

50 minutes

Instructions: No notes, texts, calculators, or other aids are permitted.
Be sure to show all calculations.

1. a) State the "squeeze rule" for the convergence of a sequence.
 b) Find and verify the limits of sequences with the general terms
- (i) $a_n = \frac{n^{2/3} \sin(n!)}{n+1}$,
- (ii) $a_n = \sqrt{n+1} - \sqrt{n}$.

2. Use the integral test to prove that

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^s}$$

converges if $s > 1$ and diverges if $0 < s \leq 1$.

3. Using either the ratio or root tests, evaluate the convergence or divergence of :

a) $\sum_{n=3}^{\infty} \frac{1}{(\ln n)^n}$, b) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

4. Assess the convergence or divergence of the following series by any suitable method :

a) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+100}$,

b) $\sum_{n=1}^{\infty} \frac{1+\sqrt{n}}{(n+1)^3-1}$,

c) $\sum_{n=1}^{\infty} e^{-n^2}$.