Math 231 Group Lab

Lab Number 1 Lab Date 09/06/06

Student Name: Lara Meyer

Group Name: Santa's Little Elves

16/20

Problem 1

Evaluate $\int_0^1 x\sqrt{1-x^4}\,dx$

$u = x^2$ ✓

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

6/10

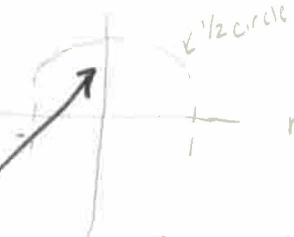
are these the correct limits in terms of u? justify. -2

$\frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du$ ✓

$\frac{1}{2}\int_0^1 (1-u^2)^{1/2}\,du$

$= \frac{1}{2}\left(\frac{1}{4}\pi\right)$

$= \frac{1}{8}\pi$



$A_{\text{semi circle}} = \frac{1}{2}\pi(1)^2$

$A = \frac{1}{4}\pi$

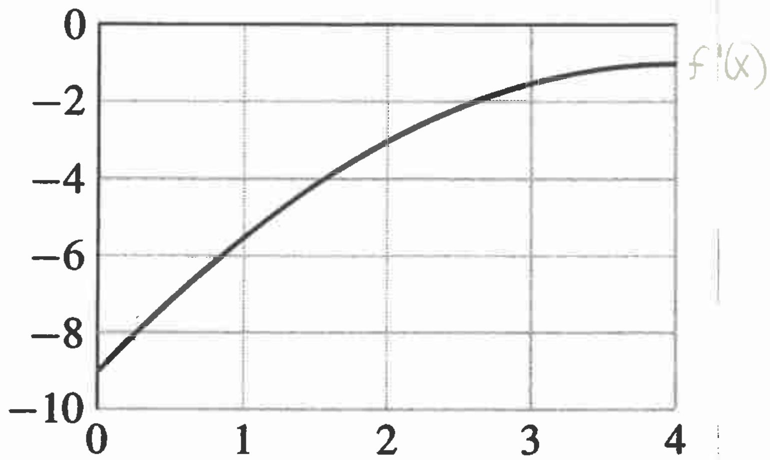
⇓

¼ circle

why does this translate into a circle? -2

Problem 2

f is an antiderivative of the function shown below



Suppose that $f(0) = 0$ and that $I = \int_0^4 x f(x)\,dx$. Rank the values L_n, R_n, and I in increasing order.

$R_n \le I \le L_n$

10/10

If $f'(x)$ is negative on $[0,4]$, $f(x)$ must be decreasing so $f(x)$ is monotone
Because the slope of $f'(x)$ is always increasing /positive
$f''(x)$ must be positive therefore $f(x)$ must be concave
up. B/c $f(0) = 0$ + $f(x)$ is always decreasing on the interval
$[0,4]$, then $x f(x)$ will always be negative decreasing therefore

$R_n \le I \le L_n$

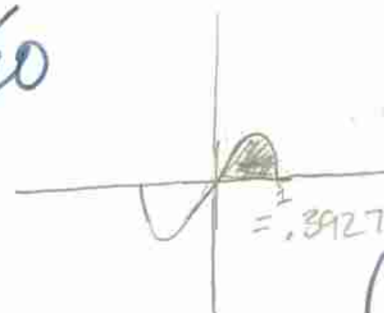
12/20

Math 231 Group Lab

Lab Number 1 Lab Date 9/6/06

Student Name: Erika Nelson

Group Name: DREM



$\frac{6}{10}$

Problem 1

Evaluate $\int_0^1 x\sqrt{1-x^4}\,dx$

$\boxed{\doteq .3927}$

$u = x^2$ ✓

$du = 2x\,dx$

$\frac{1}{2}\int_0^1 (1-u^2)^{1/2}\,du$ ✓

$\int_0^1 x(1-x^4)^{1/2}$

$\frac{1}{2}\left(\frac{1}{4}\pi r^2\right)$

$\frac{1}{2}\left(\frac{1}{4}\pi(1)^2\right) = \frac{1}{8}\pi = \frac{\pi}{8} \approx .3927$

$(1-u^2)^{1/2}$ = half of a circle

$r=1$

are these the correct limits in u? Justify? (-2)

why? I need more than a graph! (-2)

$(x f(x))' = f(x) + x f'(x)$

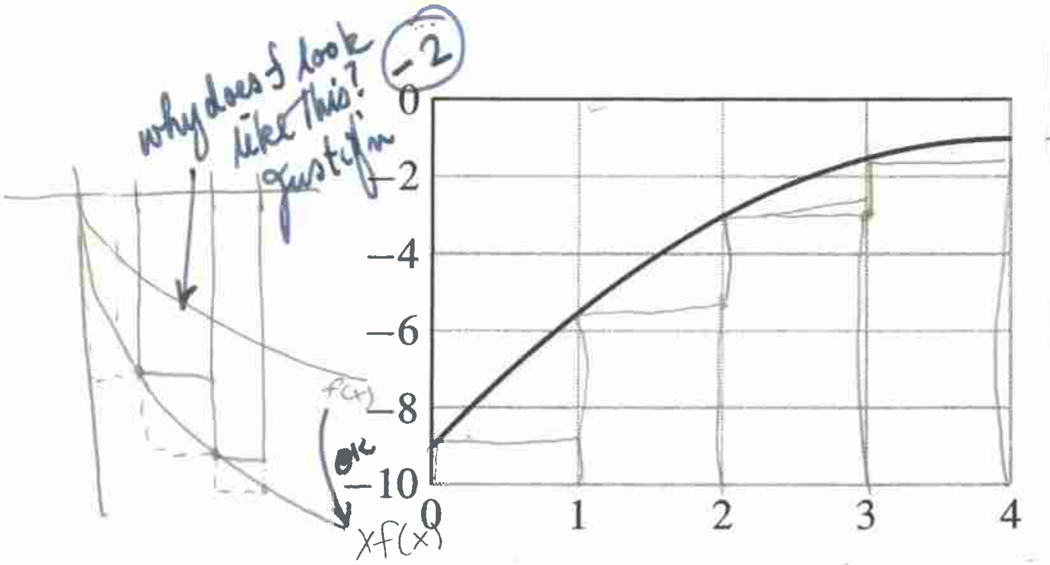
Problem 2

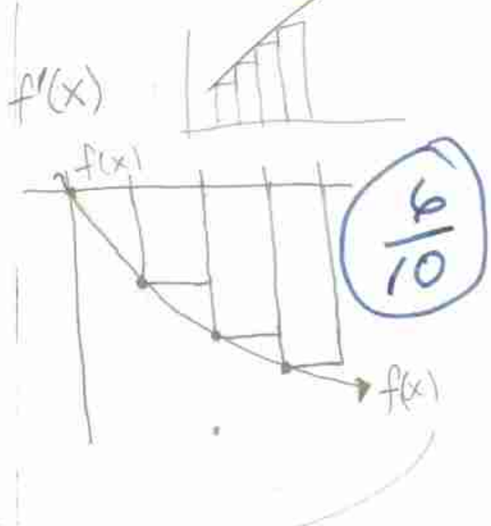
f is an antiderivative of the function shown below

how are you using this?

why does f look like this? justif'n

-2





$\frac{6}{10}$

Suppose that $f(0) = 0$ and that $I = \int_0^4 x f(x)\,dx$. Rank the values L_n, R_n, and I in increasing order.

always neg.
always dec.

why? -1

$L_n \rightarrow$ underestimate

$R_n \rightarrow$ overestimate

-1 This doesn't agree with

why increasing?

$\boxed{R_n \leq I \leq L_n}$ ✓

Not enough justif'n

12/20

Math 231 Group Lab

Lab Number 1 Lab Date 09-06-06

Student Name: Richard Ou

Group Name: New kids on the block

Problem 1

Evaluate $\int_0^1 x\sqrt{1-x^4}\,dx \cdot \frac{\sqrt{1-x^4}}{\sqrt{1-x^4}}$

$\sin^{-1} x = \frac{x'}{\sqrt{1-x^2}}$

$\int \frac{x(1-x^4)}{\sqrt{1-x^4}} \to \frac{1}{2}\int \frac{2x(1-x^4)}{\sqrt{1-(x^2)^2}}$

$u = (1-x^4)$ $dv = \frac{2x}{\sqrt{1-(x^2)^2}}$

$du = -\frac{1}{4}x^3$ $v = \sin^{-1}(x^2)$

$\int u\,dv = uv - \int v\,du$

$(\sin^{-1}(x^2))(1-x^4) - \int$

$\frac{2x}{4\sqrt{1-x^4}} + \frac{1}{4}\left(2x\sqrt{1-x^4} - \frac{2x^5}{\sqrt{1-x^4}}\right)$

$\left[\frac{\sin^{-1}(x^2) + x^2\sqrt{1-x^4}}{4}\right]_0^1$

$\frac{\pi}{8}$

$u = \sqrt{1-x^4}$ $dv = x$

$du = \frac{-2x^3}{\sqrt{1-x^4}}$ $v = \frac{1}{2}x^2$

$\frac{1}{2}x^2\sqrt{1-x^4} - \int\left(\frac{1}{2}x^2\right)\left(\frac{-2x^3}{\sqrt{1-x^4}}\right)$

$+\int \frac{x^5}{\sqrt{1-x^4}}$

$\frac{1}{2}\int \frac{2x}{\sqrt{1-(x^2)^2}}$

$\frac{1}{2}\sin^{-1}(x^2)$

$\frac{7}{10}$

Which sol'n am I to grade? -1

$u = x^2$ ✓ $du = 2x\,dx$ ✓

$\frac{1}{2}\int \sqrt{1-u^2}\,du \to y = \sqrt{1-u^2} \to y^2 + u^2 = 1$ ✓

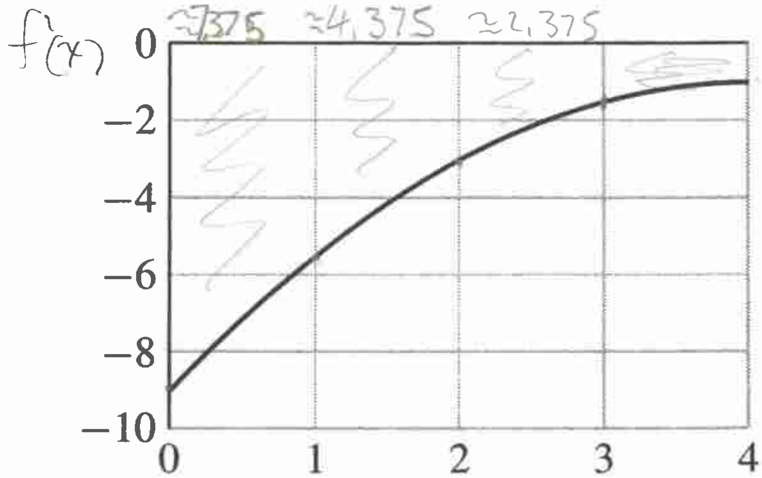
$\to \frac{\pi}{4}$

no limits! -2

$\frac{\pi}{8}$ ✓

Problem 2

f is an antiderivative of the function shown below



5/10

Suppose that $f(0) = 0$ and that $I = \int_0^4 x f(x)\, dx$. Rank the values L_n, R_n, and I in increasing order.

I is a constant! (Notation!)

$\cancel{X} = xf(x)$

$I'' = = xf'(x) + f(x)$

$(xf(x))' = xf'(x) + f(x)$

derivitive is neg

deriv of what? -1

$f'(x) = -x^2$ X

$f(x) = \frac{-1}{3}x^3$ X

I is decreasing because the slope is negative

$\therefore R_n \leq I \leq L_n$

how do you know this? -2

$f'(x) = .5x^2 + 4x - 9$

$f(x) = \frac{x^3}{6} + 2x^2 - 9x$ X

$I' \approx -\frac{x^4}{3}$

Have you used $f(0) = 0$? -2

16/20

Math 231 Group Lab
Lab Number 1 **Lab Date** 9/6/06
Student Name: Maggie Schneider
Group Name: Calculus Girls

Problem 1

Evaluate $\int_0^1 x\sqrt{1-x^4}\,dx$

are these the correct limits in u? Justify. -2

$u = x^2$
$du = 2x$

$\frac{1}{2}\int_0^1 \sqrt{1-u^2}\,du$ - graph is a quarter circle

8/10

$y = \frac{\sqrt{1-u^2}}{4} = y^2 + u^2 = 1$
circle ✓

equation of a circle
$x^2 + y^2 = r^2$
$r = 1$

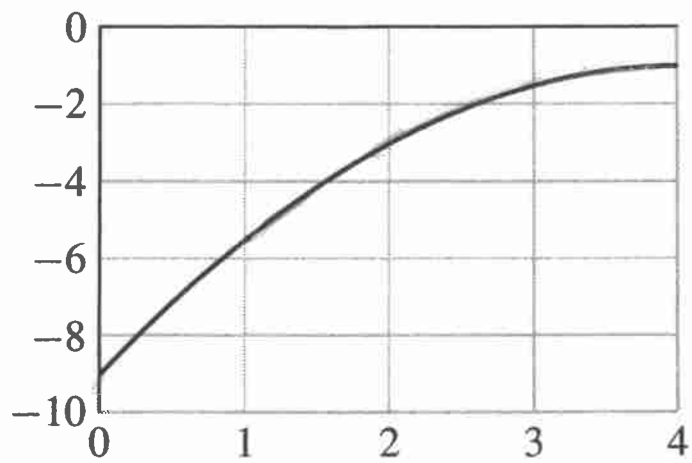
circle $= \pi r^2$

quarter circle $= \frac{\pi r^2}{4} = \frac{\pi}{4}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{8}}$ ✓

Problem 2

f is an antiderivative of the function shown below

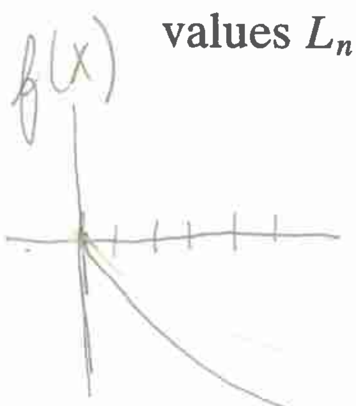
$f(x)$ - dec
conc up



$f'(x)$

8/10

Suppose that $f(0) = 0$ and that $I = \int_0^4 x f(x)\,dx$. Rank the values L_n, R_n, and I in increasing order.



$(xf(x))' = xf'(x) + f(x)$

$(+)(-) + (-) = \boxed{-}$

✓pos b/c x is btw 0 & 4 — neg b/c f(x) is always below the x-axis — negative (look at graph)

where did you get the graph from?? Justify! (−1)

The function is always neg so antideriv is decreasing on [0,4]

*Since $\int_0^4 x f(x)$ is always neg, then the antiderivative will always be decreasing. When it's always monotone, specifically decreasing, R_n will be less than L_n.

the integral is a constant!

not sure what this means! (−1)

L_n: extra

R_n: less

$\boxed{R_n \leq I \leq L_n}$ ✓