

Math 231: Calculus II

Study guide for midterm 2 (mastery exam)

The second midterm will cover material from chapters 7 and 8. The goal of the exam is to test basic skills and problem solving WITHOUT THE USE OF A CALCULATOR. The following is a list of skills that may be evaluated in this test:

- Differentiation Formulas:

$$\begin{array}{lll} \frac{d}{dx}(u^n) = nu^{n-1}u' & \frac{d}{dx}(uv) = uv' + u'v & \frac{d}{dx}(u+v) = u' + v' \\ \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2} & \frac{d}{dx}(\sin u) = u' \cos u & \frac{d}{dx}(\cos u) = -u' \sin u \\ \frac{d}{dx}(\sec u) = u' \sec u \tan u & \frac{d}{dx}(\tan u) = u' \sec^2 u & \frac{d}{dx}(e^u) = u' e^u \\ & \frac{d}{dx}(\ln u) = \frac{u'}{u} & \end{array}$$

- Integration (+C omitted)

$$\begin{array}{lll} \int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \\ \ln x & \text{if } n = -1 \end{cases} & \int \sin x dx = -\cos x & \int \cos x dx = \sin x \\ \int \sec^2 x dx = \tan x & \int \tan x dx = \ln |\cos x| & \int e^x dx = e^x \\ \int \sec x \tan x dx = \sec x & \int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) & \int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a} \arcsin\left(\frac{x}{a}\right) \\ & \int u dv = uv - \int v du & \end{array}$$

- Area. Consider two functions $f(x) \geq g(x)$. The area bound by the graphs of f and g between $x = a$ and $x = b$ is

$$\text{Area} = \int_a^b (f(x) - g(x)) dx$$

If you are only given the two curves, you must find their points of intersection (a and b) and determine which lies above the other on the interval $[a, b]$. Then you integrate as above.

- Volume. Remember that Volume is the antiderivative of the cross-sectional Area function. So finding the volume amounts to finding this area function $A(x)$ and integrating over the appropriate values of a and b . That is

$$\text{Volume} = \int_a^b A(x) dx$$

- Arc Length. The arc length of a curve $y = f(x)$ in the interval $[a, b]$ is given by

$$\int_a^b \sqrt{1 + (f'(x))^2} dx$$

- Differential Equations. Given a differential equation, you are expected to be able to determine if Separation of Variables is possible, and if so, you are expected to be able to find the family of functions which are solutions of the Differential Equation (with some undetermined constant). If we have an initial value, then it is an IVP and you are expected to find the solution to the IVP. This could be an implicit solution, and if feasible, you should be able to give the explicit solution, that is solve for y as a function of t .
- Integration by Parts. You are expected to recognize the necessity of integrating by parts, remember the procedure to do so, and be able to choose the appropriate u and dv to carry out the evaluation.
- Rational Functions and Partial Fractions. You are expected to be able to integrate simple rational functions. You should be able to recognize whether a rational function is proper or improper, and reduce it to a sum of integrable proper functions. You are expected to know how to use the partial fraction decomposition to accomplish this.
- Trigonometric Antiderivatives. You are expected to be able to solve common integrals of trigonometric functions and their products, and know how to use trigonometric substitutions to solve integrals involving roots of quadratic expressions. You should know the common trigonometric identities and the double angle formulas.