Math 230
Mastery Test III
March 17, 2005

Name: _____

Instructor: Jeremy Tyson / Micah James

Directions:

- You may not use any books or notes.
- You may not use any kind of computing device (calculator, computer, etc.).
- Do all problems in the space provided.
- Show **ALL** work. Make sure that your work is legible and neatly ordered. Credit will **NOT** be given if you only give the final answer.

Score:

1. _____ / 20

2. _____ / 15

3. _____ / 15

4. _____ / 15

5. _____ / 15

6. _____ / 20

TOTAL. _____ / 100

1. (20 points) Let $f(x) = \ln(1+x)$ and let $I = \int_1^5 f(x)\,dx$.

(a) Explain why $|f'(x)| \leq \frac{1}{2}$ and $|f''(x)| \leq \frac{1}{4}$ for all x in $[1, 5]$.

(b) Use part (a) to find error bounds for R_{10} and M_5.

2. (15 points) Consider the IVP $y' = 1 + xy$ with $y(0) = 1$.

(a) Use Euler's method with one step of size 4 to estimate $y(4)$.

(b) Use Euler's method with four steps of size 1 to estimate $y(4)$.

3. (15 points) Find the arc length of the graph of the function $y = \frac{1}{3}x^{3/2}$ between $x = 4$ and $x = 8$.

4. (15 points) Sketch the region bounded by the graphs of $y = x^3$, $y = 16 - x^3$, $x = 0$ and $x = 2$. Find the volume of the solid obtained by revolving this region about the x-axis.

5. Find the solution to the IVP $y' = xy^2$, $y(0) = 1$. Give your answer in the form $y = f(x)$.

6. (20 points) (a) Find $\int xe^{3x}\,dx$. Check your answer by differentiation.

(b) Find $\int e^{x}\sin x\,dx$. Check your answer by differentiation.