Math 285 - Differential Equations - Test No. 1
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Instructions: You have 50 minutes to complete the examination. Notes, texts, and other aids are not permitted in the examination, but calculators may be used. Complete answers are required for full credit. Good Luck!

Theory

1. (2 points) For the initial value problem $(x-1)(x-3)y' + x^2 y = x-1, \;\; y(2) = 0,$ find an interval in which the solution exists and is unique.

2. (3 points) Find a region in the *xy* plane where the solution of the initial value problem $y' = \tan(xy), \;\; y(0) = 0,$ exists and is unique. What is this solution?

3. (5 points) Select the correct answer for each of the following questions:

a) Solutions of $y' = x/y$ and $y' = y/x$ intersect at right angles.
A. yes B. no Answer: _____

b) The order of the differential equation $y' + \sin(y'') = 1$ is
A. 0 B. 1 C. 2 Answer: _____

c) The *definition* of exactness for a differential equation $M(x,y) + N(x,y)y' = 0$ is that
A. $M_y = N_x$
B. $\psi_x = M$ and $\psi_y = N$ for some function ψ
C. Neither of the above Answer: _____

d) The differential equation whose solutions are $y = cx^3$ is
A. $y' = 2y/x$
B. $y' = -3x/y$
C. $xy' = 3y$ Answer: _____

e) At $x = 2$ the slope of the solution of the initial value problem $y' + 4y = x^2$, $y(2) = -1$, is
A. 0 B. 8 C. -1 Answer: _____

Techniques and Computation

4. (5 points) Indicate the *type* of each of the following differential equations, i.e. are they
A. exact B. linear C. separable D. homogeneous or E. none of these ?

a) $\sqrt{x+y} + y' = 0$ Answer: _____

b) $(x-1)y - 1 + (x^2 - 1)y' = 0$ Answer: _____

c) $e^x + ye^{xy} + (e^y + xe^{xy})y' = 0$ Answer: _____

d) $xy + 3y^2 + x^2y' = 0$ Answer: _____

e) $\log(y' + 3xy) = x$ Answer: _____

5. (10 points) Solve the initial value problem $2x + ye^{xy} + xe^{xy}y' = 0,\ y(1) = 0.$ Find the largest x interval in which the solution is defined.

6. (10 points) Solve the initial value problem $2xyy' = 4x^2 + 3y^2,\ y(1) = -1,$ and determine y explicitly in terms of x.

Thinking

7. (10 points) A body of mass m is moving with velocity v in a *gravity-free* laboratory (i.e. outer space). It is known that the body experiences resistance in its flight proportional to the square root of its velocity. Consequently the motion of the body is governed by the problem

$$m\frac{dv}{dt} = -k\sqrt{v}, \quad v(0) = v_0,$$

where k is a positive constant. Find $v(t)$. Does the body utimately come to rest? If so, when does this happen?