

**Math 285 - Differential Equations - Test No. 2**  
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*Instructions: You have 50 minutes to complete the examination. Notes, texts, and other aids are not permitted in the examination, but calculators may be used. Complete answers are required for full credit. Good Luck!*

**Techniques and Computation**

1. (10 points) Find the general solution of  $y'' - 3y' - 4y = 50xe^{-x}$ .
2. (10 points) Two solutions of the homogeneous equation  $x^2y'' + xy' - y = 0$  are  $y_1 = x$  and  $y_2 = 1/x$ . Use this information to find the general solution of

$$x^2y'' + xy' - y = \frac{99}{x^{10}}$$

3. (7 points) For power series solutions  $y = \sum_{n=0}^{\infty} a_n x^n$  of the equation

$$y''' + y = 0,$$

what is the recurrence relation?

**Thinking Theoretically**

4. (5 points) The existence and uniqueness theorem tells us that the initial-value problem

$$y'' + x^2y = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

define exactly one function  $y(x)$ . Using only the existence and uniqueness theorem, show that this function has the additional property  $y(-x) = y(x)$  by finding the initial-value problem satisfied by the new function  $Y(x) = y(-x)$ .

**Thinking**

5. (10 points) a) Find the general solution of  $y''' + y'' - y' - y = 0$ .  
b) Find the unique solution of the equation in part a) that satisfies the initial conditions  $y(0) = 0$ ,  $y'(0) = \alpha$ ,  $y''(0) = 2$  where  $\alpha$  is a constant.  
c) What is the value of  $\alpha$  if the solution approaches 0 as  $x \rightarrow \infty$  ?