NAME: ____

Math 285 - Differential Equations - Test No. 3
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Instructions: *You have 50 minutes to complete the examination. The use of notes, texts, and other aids is not permitted. Calculators may be used, but are not necessary. Complete answers are required for full credit. Good luck!*

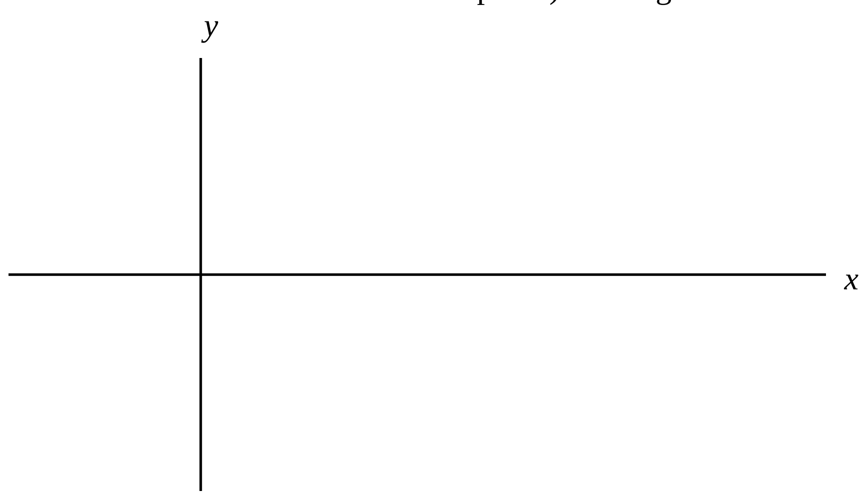
10 1. Find all separated solutions of the following boundary value problem for a modified heat equation (helpful formulas may be found on the last page):

$$u_t + u = u_{xx}\ , \quad 0 < x < \pi, \quad t > 0$$
$$u_x(t,0) = 0, \quad u_x(t,\pi) = 0.$$

7 2. a) Find the Fourier cosine series of the function

$$f(x) = \begin{cases} x & \text{if } 0 \le x < \frac{\pi}{2} \\ -1 & \text{if } \frac{\pi}{2} \le x < \pi \end{cases}$$

3 b) On the axes below draw an accurate graph in the interval $-\pi \le x \le 3\pi$ of the function to which the Fourier series in part a) converges.



10 3. Find all non-zero solutions of the endpoint problem

$$y'' + \sigma y = 0, \quad y'(0) = 0, \quad y(\pi) = 0.$$

You may restrict your attention to the case when $\sigma \ge 0$.

5 4. Show how the non-homogeneous boundary value problem

$$u_t = a^2 u_{xx}, \ \ 0 < x < 1, \ \ t > 0$$
$$u(t,0) = T, \ \ u_x(t,1) = M,$$

where T and M are constants, can be reduced to the corresponding homogeneous problem

$$u_t = a^2 u_{xx}, \quad 0 < x < 1, \quad t > 0$$
$$u(t,0) = 0, \quad u_x(t,1) = 0.$$

5 5. Demonstrate why the full Fourier series of a function $f(x)$ defined on the interval $-\ell < x < \ell$ reduces to the Fourier sine series of $f(x)$ on $0 < x < \ell$ if the function f is an odd function of x, i.e. $f(-x) = -f(x)$.

Useful Facts

$$y'' + \sigma y = 0, \quad y(0) = 0, \quad y(\ell) = 0, \quad y(x) \neq 0$$
$$\Downarrow$$
$$\sigma = \frac{n^2 \pi^2}{\ell^2}, \quad y(x) = \sin\frac{n\pi x}{l}, \quad \text{for } n = 1, 2, 3, \ldots$$

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$$y'' + \sigma y = 0, \quad y'(0) = 0, \quad y'(\ell) = 0, \quad y(x) \neq 0$$
$$\Downarrow$$
$$\sigma = 0, \quad y(x) = 1, \quad \text{and}$$
$$\sigma = \frac{n^2 \pi^2}{\ell^2}, \quad y(x) = \cos\frac{n\pi x}{l}, \quad \text{for } n = 1, 2, 3, \ldots$$

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