

Math 285 - Differential Equations - Test No. 1
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Instructions: You have 50 minutes to complete the examination. Notes, texts, and other aids are not permitted in the examination, but calculators may be used. Complete answers are required for full credit. Good Luck!

Theory

1. (8 points) Select the correct answers for each of the following questions. **Note: Some questions may have several correct answers, and each is worth a point!**

a) The order of the differential equation $y'' = 3x(y'' + 1)^2 + (y')^{22}$ is:
A. 1 B. 2 C. 3 D. 22 Answer: _____

b) Multiplying $y' + P(x)y = Q(x)$ by an integrating factor:
A. makes the equation exact
B. makes the left two terms disappear
C. turns the problem into a separable equation
D. makes the left two terms combine into a single derivative
Answer: _____

c) What initial value problem has $y(x) = e^{-x^2} \int_0^x e^{s^2} ds$ as its solution:
A. $y' = 2xy, \quad y(0) = 0$
B. $y' = -2x, \quad y(0) = 0$
C. $y' = -2xy + 1, \quad y(1) = 0$
D. none of the above
Answer: _____

d) The slope of the solution of $y' = e^{x+y}$ at the point $y(0) = 1$ is:
A. 1 B. e
C. e^e D. 0
Answer: _____

e) Solutions of $\frac{dP}{dt} = rP, \quad P(0) = P_0 > 0, \quad r > 0$ are:
A. always increasing B. always decreasing
C. first rising then falling D. none of the above
Answer: _____

f) For what initial values given below are solutions of $y' = x^{2/3} + (y+1)^{2/3}$ assured to exist but not guaranteed to be unique:
A. $y(0) = -1$ B. $y(1) = -1$
C. $y(0) = 0$ D. $y(-1) = 1$
Answer: _____

Techniques and Computation

2. (6 points) Select the correct answers for each of the following questions. **Note: Some questions may have several correct answers, and each is worth a point!**

a) The differential equation $y^3 + 2y^2xy' = 0$ is:

- A. linear
- B. variables separable
- C. homogeneous
- D. exact
- E. none of the above

Answer: _____

b) The differential equation $(x^2 + y^2) + 2xyy' = 0$ is:

- A. linear
- B. variables separable
- C. homogeneous
- D. exact
- E. none of the above

Answer: _____

c) The differential equation $(x^2 + y^2) + y' = 0$ is:

- A. linear
- B. variables separable
- C. homogeneous
- D. exact
- E. none of the above

Answer: _____

3. (12 points) For the initial value problem $y^2 \cos x + 2y \sin x y' = 0$, $y(\frac{\pi}{2}) = 1$, show that the differential equation is exact, solve it subject to the initial condition, and determine $\lim_{x \rightarrow 0} y(x)$.

4. (9 points) Solve $y' = \frac{70x}{3y^2}$, $y(1) = 2$, and find $y(0)$.

Thinking

5. (15 points) A small lake supports a population of fish which, under normal circumstances, enjoys a natural birth process with birth rate $r > 0$. However, a fishing company has just discovered the lake and is now drawing fish out of the lake at a rate of h fish per day. A model capturing this situation is:

$$\frac{dP}{dt} = -h + rP, \quad P(0) = P_0$$

- a) Find the equilibrium level P_e of fish in the lake.
- b) Find $P(t)$ explicitly (i.e. solve the initial value problem) and draw a graph of the solutions it represents for a collection of different initial conditions.
- c) Under what condition relating h , r , and P_0 is the lake ultimately fished out? What does this tell you about the equilibrium in part a)?