

Name: _____

Math 285 - Differential Equations
Test 2 - March 16, 1998

Instructions: Complete solutions are required for full credit so show all your work. You have 50 minutes to complete the exam. Good Luck!

1. (5 points) For the linear differential equation $y'' + p(x)y' + q(x)y = 0$ give a proof of the fact that if y_1 and y_2 are solutions, then $c_1y_1 + c_2y_2$ is a solution for any choice of the constants c_1 and c_2 .

2. (2 points) a) Define the term "linearly dependent" as it applies on an interval I to a collection of functions $y_1(x), \dots, y_n(x)$.
(5 points) b) Using the definition in a), show directly that the functions $y_1(x) = \sin x$, $y_2(x) = \cos x$, $y_3(x) = \sin 2x$ are linearly independent for all x .
(3 points) c) What is the Wronskian of a collection of functions and why is it important in the study of linear differential equations?
(3 points) d) Compute the Wronskian of the functions $y_1 = e^{rx}$ and $y_2 = xe^{sx}$ and determine conditions on r and s under which it is non-zero over an interval of x 's.

3. (10 points) Solve: $y^{(3)} + 9y' = 0$, $y(0) = 3$, $y'(0) = -1$, $y''(0) = 2$.

4. (12 points) Find the general solution of:

$$y'' - 3y' + 2y = 3e^{-x} - 10\cos 3x$$

5. (10 points) Find the *form* (i.e. apart from the determination of constants) of a particular solution of the equation

$$(D^2 + 4)(D - 3)^2(D^2 - 6D + 13)y = 4e^{3x} \sin 2x + x^2e^{3x}$$

You do not have to determine the constants in your answer, but you must be sure that it will work. Provide enough explanation of your steps to justify your reasoning.

6. (10 points) Solutions of the differential equation

$$x^2y'' + 2xy' - 12y = 0$$

are known to be of the form x^r where r is an unknown constant. Use this information to find the general solution. Justify your reasoning..