

\*\*\*\*\* Name: \_\_\_\_\_

**Math 285 - Differential Equations**  
**Test 3 - May 4, 1998**

**Instructions:** Complete solutions are required for full credit so show all your work. You have 50 minutes to complete the exam. Good Luck!

Note:  $\int e^{ax} \sin bx \, dx = \frac{a}{a^2 + b^2} e^{ax} \sin bx - \frac{b}{a^2 + b^2} e^{ax} \cos bx + C$

1. Find the Fourier series of the function  $f(t)$  which is defined on a whole period by

$$f(t) = \begin{cases} 2 & \text{if } -2 < t < 0 \\ -2 & \text{if } 0 < t < 2 \end{cases}$$

In particular, write out the approximation to the Fourier series represented by the first 4 non-zero terms and draw a graph on  $-4 < t < 4$  of the function to which the Fourier series converges. Does this Fourier series allow you to find the value of the series below, and, if so, what is that value:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

2. Use the method of variation of parameters to find a particular solution of the equation
- $$y'' - y = \sin x$$

3. Using the method of elimination, find the general solution of the linear system

$$x' = 2x + y, \quad y' = x + 2y - e^{2t}$$

4. Using matrix methods, find the general solution of the linear system

$$\mathbf{x}' = \mathbf{A}\mathbf{x} \quad \text{where} \quad \mathbf{A} = \begin{bmatrix} 8 & 8 & 8 \\ -4 & -4 & -4 \\ -2 & -3 & 0 \end{bmatrix}.$$

To simplify the calculation, you may use the fact that  $\det(\mathbf{A} - \lambda\mathbf{I}) = -\lambda^3 + 4\lambda^2 - 4\lambda$ .

5. The following differential equation governs the displacement  $x(t)$  from equilibrium of a body of mass  $m$  on a spring with constant  $k$  and subject to linear resistance of size  $c > 0$ . The mass is experiencing a forcing effect of the form  $F_0 \sin \omega t$ .

$$mx'' + cx' + kx = F_0 \sin \omega t$$

Find the *steady state* solution of this equation. Why is it referred to as a steady state?