Math 415 Old Exam # 3

1. (10 points)

   (a) The Gram-Schmidt procedure takes a collection of vectors v_1, v_2, v_3 and produces from them a set of orthogonal vectors w_1, w_2, w_3. Give formulas for the w_i's in terms of the v_i's

   (b) Given the polynomials $p_1(x) = 1, p_2(x) = x - 1, p_3(x) = x^2$ and the inner product $\langle p, q \rangle = \int_0^2 p(x)q(x)dx$, find the corresponding set of orthogonal polynomials given by the Gram-Schmidt procedure.

2. (10 points) Find the closest point to $b = \begin{pmatrix} 1 & 2 & -1 & 3 \end{pmatrix}^T$ in the subspace $W = \text{span}\left\{\begin{pmatrix} 1 & 0 & 2 & 1 \end{pmatrix}^T, \begin{pmatrix} 1 & 1 & 0 & -1 \end{pmatrix}^T\right\}$

3. (8 points)

   (a) State Fredholm's Criterion for solutions of $Ax = b$

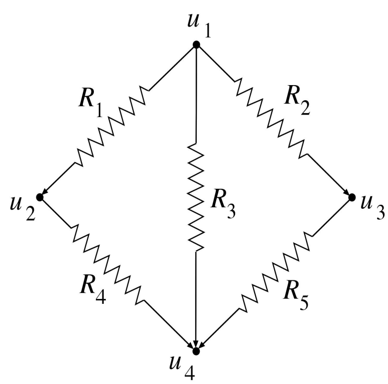
   (b) Given the following information:

$$A = \begin{pmatrix} 1 & -1 & 2 & -2 \\ 0 & 1 & -2 & 1 \\ 1 & 3 & -5 & 2 \\ 5 & -1 & 9 & -6 \end{pmatrix}, \ker A = \left\{\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}\right\}, \operatorname{co}\ker A = \left\{\begin{pmatrix} 2 \\ 24 \\ -7 \\ 1 \end{pmatrix}\right\}$$

   find those restrictions on a vector b such that $Ax = b$ has a solution. Are these satisfied for $b = \begin{pmatrix} -1 & 1 & 4 & 6 \end{pmatrix}^T$?

   (c) For the matrix in part b), what additional restrictions on x are needed in order for the solution of $Ax = b$ to be the solution of minimum norm. Explain.

4. (10 points) For the electrical network below assume that an external current of size 1 amp enters node 1 and is removed from node 4. If all the wires have resistance 7 and node 2 is grounded, find the voltage potentials at each node. Explain your work.



5. (12 points)

   (a) Explain why the function $L : R^3 \to R^3$ given by

   $$L[v] = L\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + y - z \\ x + 2y + 2z \\ -x + y + 2z \end{pmatrix}$$

   is linear. Be specific.

   (b) For the linear function

   $$L[v] = L\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4x + 3y \\ 3x + 2y \end{pmatrix}$$

   find its matrix representation relative to the satndard basis.

   (c) If the function L in part b) is invertible, find its inverse, expressing it in the same form as L

6. (10 points) Find all eigenvalues and eigenvectors of

$$A = \begin{pmatrix} 0 & 0 & 0 \\ -3 & 6 & -3 \\ -3 & 6 & -3 \end{pmatrix}$$