Math 415 - Supplementary Handout 2

Computing Coordinates When Basis Vectors Are Orthogonal

Consider the following 8 vectors that represent a basis of R^8:

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, v_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$v_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, v_6 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}, v_7 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}, v_8 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}$$

Computing the coordinates of a vector v, let us say $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}^T$ relative to a basis, that is the numbers c_i in the linear como representation

$$v = c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 + c_5v_5 + c_6v_6 + c_7v_7 + c_8v_8,$$

usually involves Gaussian elimination on the augmented matrix

$$\left(\begin{array}{cccccccc|c} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 8 \end{array}\right)$$

However, if we note that our basis vectors are orthogonal (relative to the dot product), then a much simpler calculation is possible:

$$\begin{aligned} v_1 \cdot v &= c_1v_1 \cdot v_1 + c_2v_1 \cdot v_2 + c_3v_1 \cdot v_3 + c_4v_1 \cdot v_4 \\ &\quad + c_5v_1 \cdot v_5 + c_6v_1 \cdot v_6 + c_7v_1 \cdot v_7 + c_8v_1 \cdot v_8 \\ &= c_1v_1 \cdot v_1 + c_20 + c_30 + c_40 + c_50 + c_60 + c_70 + c_80 \\ &= c_1 \left\|v_1\right\|^2 \end{aligned}$$

And so

$$c_1 = \frac{v_1 \cdot v}{\|v_1\|^2} = \frac{3}{2}$$

Similarly we have

$$c_2 = \frac{v_2 \cdot v}{\|v_2\|^2} = \frac{-1}{2}$$
$$c_3 = \frac{v_3 \cdot v}{\|v_3\|^2} = \frac{7}{2}$$
$$c_4 = \frac{v_4 \cdot v}{\|v_4\|^2} = \frac{-1}{2}$$
$$c_5 = \frac{v_5 \cdot v}{\|v_5\|^2} = \frac{11}{2}$$
$$c_6 = \frac{v_6 \cdot v}{\|v_6\|^2} = \frac{-1}{2}$$
$$c_7 = \frac{v_7 \cdot v}{\|v_7\|^2} = \frac{15}{2}$$
$$c_8 = \frac{v_8 \cdot v}{\|v_8\|^2} = \frac{-1}{2}$$

And as thay say at Staples, “That was easy”.