Math 415 Lecture 20

Sec 4.4 Data Fitting

$(t_1, y_1), (t_2, y_2), \ldots, (t_m, y_m)$

Fit a fn $y = \alpha + \beta t$ to this data. In general we can't do this, so we consider the "error" in doing this:

$$e_1 = y_1 - (\alpha + \beta t_1)$$
$$e_2 = y_2 - (\alpha + \beta t_2)$$
$$\vdots$$
$$e_m = y_m - (\alpha + \beta t_m)$$

and minimize $\sum_{i=1}^{m} e_i^2$. $\Rightarrow$ (least squares approx'n)

Set $y = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix}$, $A = \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_m \end{pmatrix}$, $x = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, $e = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{pmatrix}$

$$\Rightarrow \quad y - Ax = e$$

Minimize $\|e\| = \|Ax - y\|$

Normal Eq'ns: $(A^TA)x = A^Ty \Rightarrow x^* = (A^TA)^{-1}A^Ty$

$$A^TA = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ t_1 & t_2 & \cdots & t_m \end{pmatrix} \begin{pmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_m \end{pmatrix} = \begin{pmatrix} m & \Sigma t_i \\ \Sigma t_i & \Sigma t_i^2 \end{pmatrix}$$
$$= m \begin{pmatrix} 1 & \bar{t} \\ \bar{t} & \overline{t^2} \end{pmatrix}$$

$$A^Ty = \begin{pmatrix} \quad \end{pmatrix} \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} \Sigma y_i \\ \Sigma t_i y_i \end{pmatrix} = m \begin{pmatrix} \bar{y} \\ \overline{ty} \end{pmatrix}$$

where ....

So $\alpha + \bar{t}\beta = \bar{y}$

$\bar{t}\alpha + \overline{t^2}\beta = \overline{ty}$

$$\Rightarrow \alpha = \bar{y} - \bar{t}\beta \text{ where } \beta = \frac{\overline{ty} - \bar{t}\,\bar{y}}{\overline{t^2} - (\bar{t})^2} = \frac{\sum (t_i - \bar{t}) y_i}{\sum (t_i - \bar{t})^2}.$$

Review Example 4.10.