Math 415 Lecture 33

Sec 8.3 Eigenvector Bases

Thm: If $\lambda_1, \dots, \lambda_k$ are distinct eigenvalues of a matrix A, then the corresp eigenvectors are linearly independent.

Use induction on k: Assume $\lambda_1, \dots, \lambda_{k-1}$ are distinct and $v_1, \dots, v_{k-1}$ are linearly indep. Then

$$c_1 v_1 + \cdots + c_k v_k = 0$$
$$\Rightarrow c_1 A v_1 + \cdots + c_k A v_k = 0$$
$$\Rightarrow c_1 \lambda_1 v_1 + \cdots + c_k \lambda_k v_k = 0$$
$$\Rightarrow c_1 \lambda_k v_1 + \cdots + c_k \lambda_k v_k = 0$$

Subt: $c_1(\lambda_1 - \lambda_k) v_1 + \cdots + c_{k-1}(\lambda_{k-1} - \lambda_k) v_{k-1} + 0 = 0$

$\Rightarrow c_1(\lambda_1 - \lambda_k) = 0 \cdots = c_{k-1}(\lambda_{k-1} - \lambda_k)$

$\Rightarrow c_1 = \cdots = c_{k-1} = 0.$

$c_k v_k = 0$. But $v_k \neq 0$. So $c_k = 0$. Done.

Theorem: If A is $n \times n$ and has n ~~eig~~ distinct eigenvalues then the corresp eigenvectors form a basis of $\mathbb{R}^n$.

Read pages 407-408, 409-411 for Friday.

Language:

$$V_\lambda = \ker(A - \lambda I) = \text{eigenspace corresp to } \lambda$$

$$p_A(\lambda) = (\lambda - \lambda_1)^{m_1} (\lambda - \lambda_2)^{m_2} \cdots (\lambda - \lambda_g)^{m_g}$$

m_1 is the <u>multiplicity</u> of λ_1

We say λ is <u>complete</u> if $\dim V_\lambda$ equals the multiplicity of λ. We say A is <u>complete</u> if all of its eigenvalues are complete.

Diagonalization:

A square matrix A is <u>diagonalizable</u> if there is a non-singular matrix S and a diagonal matrix Λ such that

$$S^{-1}AS = \Lambda = \operatorname{diag}(\lambda_1, \dots, \lambda_n)$$

Significance for change of basis!

How do we find the λ's and S?

$$AS = S\Lambda$$

$$A(u_1 \cdots u_n) = (u_1 \cdots u_n) \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{pmatrix} \cancel{(u_1 \cdots u_n)}$$

$$(Au_1 \cdots Au_n) = (\lambda_1 u_1 \;\; \lambda_2 u_2 \cdots \lambda_n u_n)$$

ie $Au_i = \lambda_i u_i$ so λ_i's are eigenvalues, and u_i's are eigenvectors