Math 415

Lecture 6

Sec 1.8 (continued)

Solve a specific example, i.e. one on transparencies with

$$b = \begin{pmatrix} 0 \\ 3 \\ 1 \\ 1 \end{pmatrix} \Rightarrow c = \begin{pmatrix} 0 \\ 3 \\ 3 \\ 0 \end{pmatrix}$$

$\Rightarrow v = \frac{3}{4}$, $z = -1 + 2u + v = -\frac{1}{4} + 2u$, $x = -3y - 2z + u$
$= \frac{1}{2} - 3y - 3u$

$$\therefore \begin{pmatrix} x \\ y \\ z \\ u \\ v \end{pmatrix} = \begin{pmatrix} \frac{1}{2} - 3y - 3u \\ y \\ -\frac{1}{4} + 2u \\ u \\ \frac{3}{4} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{4} \\ 0 \\ \frac{3}{4} \end{pmatrix} + y \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + u \begin{pmatrix} -3 \\ 0 \\ 2 \\ 1 \\ 0 \end{pmatrix}$$

arbitrary.

Thm $Ax = b$, a system of $m$ linear eqns in $n$ unknowns has either

a) exactly on solution
b) infinitely many solutions
b) no solutions

} relate this to planes in 3D and intersections.

(*) "system is incompatible"
"at least one eqn is inconsistent".

all planes pass through origin.

Homogeneous systems: $Ax = 0$. Here $x = 0$ is always a solution (the trivial solution) $\Rightarrow$ such systems have either one soln or infinitely many solutions

## Sec 1.9 Determinants

Defn The determinant of a square matrix $A$ is the uniquely defined scalar satisfying

a) Adding a multiple of one row of $A$ to another doesn't change the determinant

$$\det(E_{ij}(a)A) = \det A$$

b) Interchanging two rows of $A$ changes the sign of its determinant

$$\det(P_{ij}A) = -\det A$$

c) Multiplying a row by a scalar multiplies the det by that scalar

$$\det(\operatorname{diag}(1,\dots,1,a,1,\dots 1)A) = a \det A$$

d) The det of an upper triangular matrix is the product of the diag entries

$$\det U = u_{11}u_{22}\cdots u_{nn}.$$

Note that Gaussian elimination then lets us compute determinants.

Example: $A = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -3 & 4 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -4 & 2 \end{pmatrix}$

$\det(A)$

$$\rightarrow \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & -2 & 3 \\ 0 & 1 & -3 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -2 & -4 \end{pmatrix}$$

$\det(A)$ $\qquad\qquad$ $\det(A)$

$$\rightarrow \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & -4 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$-\det(A)$

so $-\det(A) = 1 \cdot 1 \cdot (-2) \cdot 3$

or $\det(A) = 6$.

Note: $A$ is nonsing iff $\det A \neq 0$. (Why?)

Properties of Determinants:

a) $\det(AB) = \det(A)\det(B)$ (even though $A$ & $B$ don't commute!)

b) $\det(A^{-1}) = \dfrac{1}{\det A}$ why?

c) $\det(A^T) = \det A$

Formulas for det:

$$\det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11}\det\begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{12}\det\begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix}$$

$$+ a_{13}\det\begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

$= \cdots =$

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