

1. (6 points) Our textbook often refers to a four-step process suggested by George Polya for problem-solving. Which one of the following choices correctly lists these four steps?

(a) studying the problem, trying different approaches, deciding a good approach, recording your work

(b) understanding the problem, devising a plan, carrying out the plan, looking back

(c) researching the problem, locating a solution, writing down the solution, turning in the solution

(d) sketching a graph, introducing variables, using algebra, comparing the answer with specific numbers

(e) analyzing the problem, drawing a picture, estimating a likely answer, calculating the correct answer

2. (8 points) For the month of June, Jared has agreed to switch from the Subway diet to the Ben & Jerry's diet. On this new diet, he will have:

- 1 scoop of ice cream on June 1,
- 2 scoops of ice cream on June 2,
- 3 scoops of ice cream on June 3,
- 4 scoops of ice cream on June 4,
- ⋮
- 29 scoops of ice cream on June 29
- 30 scoops of ice cream on June 30.

What is the total number of scoops of ice cream that Jared will have in the month of June? Be sure to simplify your answer – do not leave it as a sum or product.

$$1 + 2 + 3 + 4 + \dots + 29 + 30$$
$$= \frac{30 \cdot 31}{2} = \frac{930}{2} = 465$$

3. (6 points) Carefully explain how a chef can use a 7-minute hourglass and an 10-minute hourglass to time vegetables that must steam for 13 minutes?

- ① start both timers together
- ② when 7-minute timer finishes, put vegetables in steamer
- ③ when 10-minute timer finishes, 3 minutes later, flip it over for an additional 10 minutes.
- ④ $3 + 10 = 13$ minutes of steaming

4. (7 points) Circle **arithmetic** if the sequence could be arithmetic, circle **geometric** if the sequence could be geometric, and circle **neither** if it is impossible for the sequence to be either arithmetic or exponential. There may be more than one sequence of each type. You do not need to find a formula for the terms of any of the sequences.

- (a) arithmetic, geometric, neither 7, 14, 28, 56, 112, ... multiply by 2 to obtain next term
- (b) arithmetic, geometric, neither 9, 15, 21, 27, 33, ... add 6 to obtain next term
- (c) arithmetic, geometric, neither 4, 5, 7, 8, 10, ...
- (d) arithmetic, geometric, neither 4, 8, 12, 16, 20, ... add 4 to obtain next term
- (e) arithmetic, geometric, neither 4, 20, 40, 200, 400, ...
- (f) arithmetic, geometric, neither 2, 22, 2222, 22222222, 222222222222222, ...
- (g) arithmetic, geometric, neither 1, 2, 8, 64, 1024, ...

5. (6 points) Let a_n be the n th term in the arithmetic sequence $a_1, a_2, a_3, a_4, a_5, a_6, \dots$ below. Determine a formula for a_n .

7, 12, 17, 22, 27, 32, ...

$$a_n = 7 + 5(n-1)$$

$$a_n = 5n + 2$$

6. (16 points) Compute and simplify the following quantities.

$$\begin{aligned} \text{(a)} \quad 4^{31} \div (4^{28} + 4^{28} + 4^{28} + 4^{28}) &= 4^{31} \div (4 \times 4^{28}) \\ &= 4^{31} \div 4^{29} \\ &= 4^2 \\ &= 16 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 4 \times 9753 \times 25 &= 4 \times 25 \times 9753 \\ &= 100 \times 9753 \\ &= 975300 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 3486 + 475 + 378 + 14 + 25 &= (3486 + 14) + (475 + 25) + 378 \\ &= 3500 + 500 + 378 \\ &= 4000 + 378 \\ &= 4378 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 60 \div 2 \times 3 + 4 \times 10^2 &= 60 \div 2 \times 3 + 4 \times 100 \\ &= 30 \times 3 + 4 \times 100 \\ &= 90 + 4 \times 100 \\ &= 90 + 400 \\ &= 490 \end{aligned}$$

7. (3 points) The equation $(7 + 9) + 3 = 3 + (7 + 9)$ is an example of which one of the following properties of whole numbers?

- (a) identity property for addition
- (b) closure property for addition
- (c) commutative property for addition
- (d) associative property for addition
- (e) distributive property for multiplication over addition

8. (3 points) The equation $(2 + 6) + 5 = 2 + (6 + 5)$ is an example of which one of the following properties of whole numbers?

- (a) identity property for addition
- (b) closure property for addition
- (c) commutative property for addition
- (d) associative property for addition
- (e) distributive property for multiplication over addition

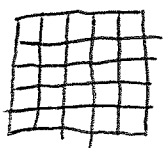
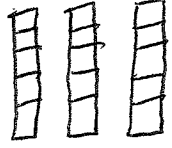
9. (3 points) The equation $9 + 7 = 7 + 9$ is an example of which one of the following properties of whole numbers?

- (a) identity property for addition
- (b) closure property for addition
- (c) commutative property for addition
- (d) associative property for addition
- (e) distributive property for multiplication over addition

10. (7 points) Convert the base ten number 40 to base five.

$$40 = 1 \times 5^2 + 3 \times 5^1 + 0 \times 5^0$$

$40_{\text{ten}} = 130_{\text{five}}$

( )

1 flat 3 longs 0 units

11. (6 points) Compute the sum of the base five numbers below. Leave your answer in base five.

$$\begin{array}{r}
 33112233_{\text{five}} \\
 + 33332211_{\text{five}} \\
 \hline
 12144444_{\text{five}}
 \end{array}$$

12. (7 points) The base ten number N is written in base five as 44332211_{five} . How do you write the base ten number $25 \times N$ in base five?

$$\begin{aligned}
 & 5^2 \times (4 \times 5^7 + 4 \times 5^6 + 3 \times 5^5 + 3 \times 5^4 + 2 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 1 \times 5^0) \\
 &= 4 \times 5^9 + 4 \times 5^8 + 3 \times 5^7 + 3 \times 5^6 + 2 \times 5^5 + 2 \times 5^4 + 1 \times 5^3 + 1 \times 5^2 + 0 \times 5^1 + 0 \times 5^0 \\
 &= 4433221100_{\text{five}}
 \end{aligned}$$

or since $25_{\text{ten}} = 100_{\text{five}}$,

$$\begin{array}{r}
 44332211 \\
 \times \quad 100 \\
 \hline
 4433221100_{\text{five}}
 \end{array}$$

13. (6 points) What digit d will result in the 7-digit number $361811d$ being divisible by 6?

$361811d$ must be divisible by both 2 and 3

$$2 \mid 361811d \Rightarrow d = 0, 2, 4, 6, \text{ or } 8$$

$$3 \mid 361811d \Rightarrow 3 + 6 + 1 + 8 + 1 + d = 20 + d$$

is a multiple of 3

$$\Rightarrow d = 1, 4, \text{ or } 7$$

Thus $d = 4$

14. (16 points) Circle T for true or F for false for each of following statements.

(a) T or F: If $d \mid m$ and $d \mid n$, then $d \mid (m+n)$

$$\left. \begin{array}{l} d \mid m \Rightarrow m = d \times c_1 \\ d \mid n \Rightarrow n = d \times c_2 \end{array} \right\} \Rightarrow m+n = d \times c_1 + d \times c_2 = d \times (c_1 + c_2) \Rightarrow d \mid (m+n)$$

(b) T or F: 2 is divisible by 10

There is no whole number c such that $2 = 10 \times c$

(c) T or F: All positive integers divide 0

Let n be a positive integer.
 $n \mid 0$ since $0 = n \times 0$

(d) T or F: 5556667770 is divisible by 4

False since 70 is not divisible by 4

(e) T or F: 9 is a divisor of 999977772222

True since $9+9+9+9+7+7+7+7+2+2+2+2 = 9 \times 8$
is a multiple of 9

(f) T or F: 11 is a factor of 929292929

False since $9+9+9+9+9 - (2+2+2+2) = 45 - 8 = 37$ is not a multiple of 11.

(g) T or F: 0 is a multiple of 4

True since $4 \times 0 = 0$

(h) T or F: 3 is a multiple of 0

False since $n \times 0 \neq 3$ for any n