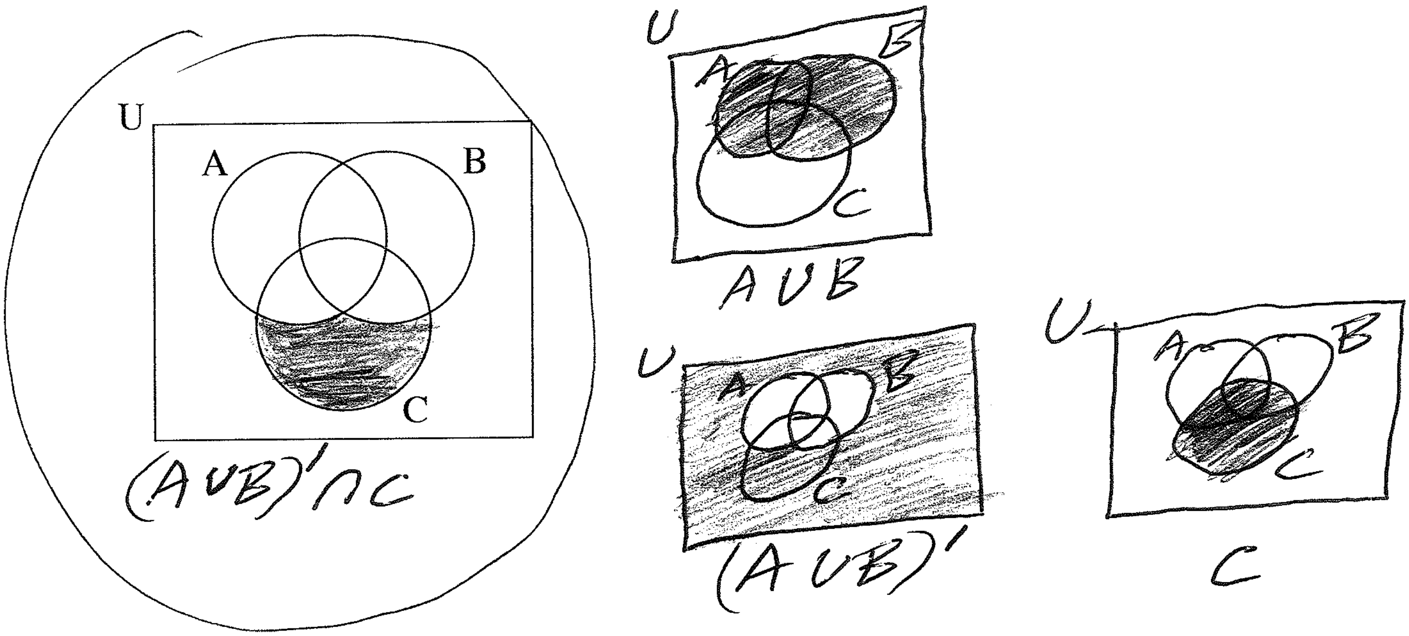
1. (7 points) Using the Venn diagram below, shade in the portion which represents the set $(A \cup B)' \cap C$.




2. (7 points) If $U = \{1, 2, 3, 4, 5, 6, 7\}$ is the universal set and $A$ and $B$ are sets satisfying $(A \cup B)' = \{1, 2, 3, 4\}$, what is the set $A' \cap B'$ ?

By De Morgan's Law, $(A \cup B)' = A' \cap B'$

so $A' \cap B' = \{1, 2, 3, 4\}$

3. (7 points) In this problem, the universal set is the set of all positive integers,

$A = \{x | x \text{ is a multiple of } 4\} = \{4, 8, 12, 16, 20, 24, \ldots\}$

$B = \{x | x \text{ is a multiple of } 6\} = \{6, 12, 18, 24, \ldots\}$

$C = \{x | x \text{ is a multiple of } 8\} = \{8, 16, 24, \ldots\}$

$D = \{x | x \text{ is a multiple of } 12\} = \{12, 24, \ldots\}$

Which one of the following statements is true?

(a) $D = A \cup B$

(b) $D = A \cap B$

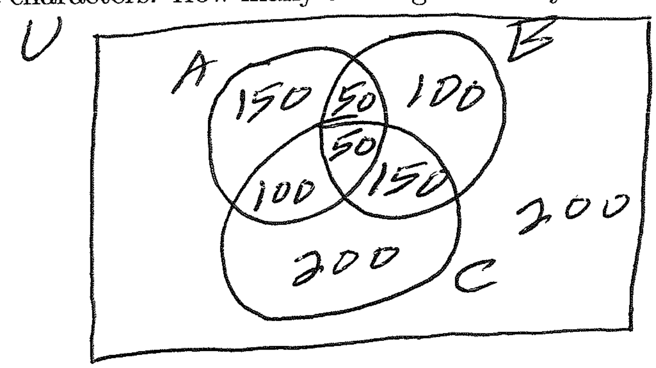
(c) $D = A \cup C$

(d) $D = A \cap C$

(e) $D = B \cup C$

(f) $D = B \cap C$

4. (7 points) A survey of 1000 young girls was taken to determine if they had ever dressed up as Ariel, Belle, or Cinderella. 350 girls had dressed as Ariel, 350 had dressed as Belle, and 500 had dressed as Cinderella. Furthermore 100 girls had dressed as Ariel and Belle, 150 had dressed as Ariel and Cinderella, and 200 had dressed as Belle and Cinderella. Finally, 50 girls had dressed as all three characters. How many of the girls surveyed had never dressed as any of these characters?




200

or can use

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$= 350 + 350 + 500 - 100 - 150 - 200 + 50$

$= 800$

so $|(A \cup B \cup C)'| = 200$

5. (7 points) Multiply the numbers 400 and $3.0 \times 10^{200}$ and write the product using scientific notation.

$400 \times 3.0 \times 10^{200}$

$= 1200 \times 10^{200}$

$= 1.2 \times 10^{203}$

6. (7 points) Francis collects spinning tops. Twelve of them are from Germany and this represents 4% of his collection. How many tops does he have in his collection?

Let $N$ be the number of tops in his collection.

4% of $N = 12$

$0.04N = 12$

$N = \frac{12}{0.04} = \frac{1200}{4} = 300$

or solve $\frac{12}{N} = \frac{4}{100}$

7. (7 points) Rewrite the expression $\dfrac{24x^{16}}{(2x^{-2})^3}$ with positive exponents and simplify.

$$\frac{24x^{16}}{\left(\frac{2}{x^2}\right)^3} = \frac{24x^{16}}{\left(\frac{8}{x^6}\right)} = 24x^{16} \cdot \frac{x^6}{8}$$

$$= 3x^{22}$$

8. (7 points) Find all solutions to the equation $(2x+3)(x-2) = -3$.

$$2x^2 - 4x + 3x - 6 = -3$$

$$2x^2 - x - 3 = 0$$

$$(2x-3)(x+1) = 0$$

$$2x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = \frac{3}{2} \quad \text{or} \quad x = -1$$

9. (7 points) Find the equation for the line which goes through the point $(4, 0)$ and is parallel to the line $y = 5x + 2$.

since it is parallel, it will also have slope 5.

$$y - 0 = 5(x - 4)$$

$$y = 5x - 20$$

10. (7 points) If a standard 6-sided die is rolled 2 times, what is the probability that you obtain at least one 6?

$1 - P(\text{no } 6\text{'s}) = 1 - \frac{5}{6} \cdot \frac{5}{6} = 1 - \frac{25}{36}$

$= \frac{11}{36}$

or look at 36 equally likely outcomes in sample space

$\left\{ \begin{array}{l} 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, \\ 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, \\ 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66 \end{array} \right\}$

11. (6 points) Evaluate the following quantities. Simplify your answers.

(a) 4! $= 4 \cdot 3 \cdot 2 \cdot 1 = 24$

(b) $\frac{100!}{98!} = \frac{100 \cdot 99 \cdot 98!}{98!} = 100 \cdot 99 = 9900$

12. (7 points) Tom and Sarah each roll a standard 6-sided die. If the value on Tom's die is at least twice the value on Sarah's die, then Tom pays Sarah \$7. If the value on Tom's die is less than twice the value on Sarah's die, then what should Sarah pay Tom in order to make this a fair game?

Tom's expected winnings per roll are

$P(\text{Tom's roll} \geq 2 \cdot \text{Sarah's roll}) \cdot (-7)$

$+ P(\text{Tom's roll} < 2 \cdot \text{Sarah's roll}) \cdot X$

$= P(61, 62, 63, 51, 52, 41, 42, 31, \text{or } 21) \cdot (-7)$

$+ P(\text{any other outcome}) \cdot X$

$= \frac{9}{36} \cdot (-7) + \frac{27}{36} X$

To be a fair game we get $\frac{9}{36}(-7) + \frac{27}{36} X = 0$

$X = \frac{63}{27} = \frac{7}{3}$ dollars or approximately \$2.33

13. (8 points) Answer the following questions. You do not need to simplify your answers.

(a) How many 5-digit positive integers are there? (No leading zeroes allowed.)

$$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$

(b) How many 5-digit positive integers have distinct digits?

$$9 \cdot 9 \cdot 8 \cdot 7 \cdot 6$$

14. (9 points) A club has 25 members (10 men and 15 women). Answer the following questions. You do not need to simplify your answers.

(a) How many ways can the club choose 5 of its members to serve in equal capacity on a committee?

$$_{25}C_5 = \frac{25!}{20!5!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

(b) How many ways can the club choose 5 of its members to serve in equal capacity on a committee if the committee must consist of precisely three men and two women?

$$({}_{10}C_3) \cdot ({}_{15}C_2) = \frac{10!}{7!3!} \cdot \frac{15!}{13!2!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} \cdot \frac{15 \cdot 14}{2 \cdot 1}$$

(c) How many ways can the club choose 5 of its members to serve in equal capacity on a committee if the committee must consist of at least one man?

TOTAL NUMBER OF 5-member committees

− NUMBER OF 5-member committees with no men

$$= {}_{25}C_5 - {}_{15}C_5 = \frac{25!}{20!5!} - \frac{15!}{10!5!}$$

could also find

$$({}_{10}C_1)({}_{15}C_4) + ({}_{10}C_2)({}_{15}C_3) + ({}_{10}C_3)({}_{15}C_2) + ({}_{10}C_4)({}_{15}C_1) + ({}_{10}C_5)({}_{15}C_0)$$