

## Probability Worksheet # 1

1. Slips of paper with the names Lois, Mary, John, Tim, Bob, Eileen and Nancy are placed in a hat. One slip is picked at random.

(a) What is the probability that a girl's name is picked? 4/7

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\}$$

(b) What is the probability that a name with 4 letters is picked?

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\} \quad 3/7$$

(c) What is the probability that a four-letter girl's name is picked?

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\} \quad 2/7$$

(d) What is the probability that a name is picked which has four letters or is a girl's name?

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\} \quad 5/7$$

(e) What is the probability that a name with more than 3 letters is picked?

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\} \quad 5/7$$

(f) What is the probability that a name with at least 3 letters is picked?

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\} \quad 7/7 = 1$$

(g) What is the probability that a name with less than 8 letters is picked?

$$S = \{\underline{\text{Lois}}, \underline{\text{Mary}}, \underline{\text{John}}, \underline{\text{Tim}}, \underline{\text{Bob}}, \underline{\text{Eileen}}, \underline{\text{Nancy}}\} \quad 7/7 = 1$$

2. A box contains 2 blue marbles and 2 yellow marbles. An experiment consists of reaching in and randomly taking two of the marbles.

(a) What is the sample space for this experiment. Are the outcomes equally likely?

$$S_1 = \{BB, BY, YB, YY\} \quad \text{not equally likely}$$

$$\text{or } S_2 = \{B_1, B_2, B_1, Y_1, B_1, Y_2, B_2, Y_1, B_2, Y_2, Y_1, Y_2\} \quad \text{equally likely}$$

→ NOTE BY means 1 BLUE AND 1 YELLOW

(b) What is the probability that both marbles will be yellow?

$$\left(\frac{1}{6}\right) \text{ AS SEEN FROM } S_2$$

$$\text{or } \frac{2}{4} \cdot \frac{1}{3} = \frac{2}{12} = \frac{1}{6}$$

- (c) What is the probability of obtaining at least one blue marble? How is this probability related to the probability found in part (b)?

$\frac{5}{6}$  AS SEEN FROM  $S_2$

$$P(\text{at least 1 blue}) = 1 - P(\text{no blues}) \\ = 1 - P(\text{both yellow}) = 1 - \frac{1}{6} = \frac{5}{6}$$

- (d) What is the probability of obtaining a blue marble and a yellow marble?

$\frac{4}{6} = \left(\frac{2}{3}\right)$  AS SEEN FROM  $S_2$

$$\text{OR } \frac{2}{4} \cdot \frac{2}{3} + \frac{2}{4} \cdot \frac{2}{3} = \frac{2}{3}$$

B Y                  Y B

3. If a fair coin is flipped 8 times, what is the probability that you obtain all 8 heads?

$$\frac{1}{2^8} = \frac{1}{256}$$

4. If a standard 6-sided die is rolled 3 times, what is the probability that you obtain all 6's?

$$\frac{1}{6^3} = \frac{1}{216}$$

5. Suppose events  $A$  and  $B$  are disjoint (i.e., mutually exclusive), what is  $P(A \cap B)$ ?

$A, B$  disjoint  $\Rightarrow A \cap B = \emptyset$

$$\text{SO } P(A \cap B) = 0$$

6. For two events  $A$  and  $B$ , if  $P(A \cup B) = \frac{2}{3}$ ,  $P(A) = \frac{1}{4}$  and  $P(B) = \frac{1}{2}$ , then what is  $P(A \cap B)$ ?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{2}{3} = \frac{1}{4} + \frac{1}{2} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{4} + \frac{1}{2} - \frac{2}{3} = \frac{3}{12} + \frac{6}{12} - \frac{8}{12} = \frac{1}{12}$$

7. Consider a regular deck of 52 cards with 13 cards (including 3 face cards) in each of 4 suits. Use the following events to determine the requested probabilities.

- E: Selecting a face card (jack, queen, king)
- F: Selecting an ace
- G: Selecting a spade
- H: Selecting a heart

- (a)  $P(E) = 12/52$   
 (b)  $P(F) = 4/52$   
 (c)  $P(G) = 13/52$   
 (d)  $P(H) = 13/52$   
 (e)  $P(E \cup F) = 16/52$   
 (f)  $P(F \cup H) = 16/52$   
 (g)  $P(G \cup H) = 26/52$   
 (h)  $P(E \cup H) = 22/52$   
 (i)  $P(E \cap H) = 3/52$   
 (j)  $P(G \cap E) = 3/52$

8. The numbers 2, 3, 4, 5 and 6 are written on five cards and placed in a hat. One card is randomly selected and its number will be used as the numerator of a fraction. A second card is randomly selected from the remaining cards and its number will be used as the denominator of the fraction. What is the probability that the fraction is greater than 1 and less than 1.5?

outcomes:  $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}, \frac{3}{2}, \frac{3}{4}, \frac{3}{5}, \frac{3}{6},$   
 $\frac{4}{2}, \frac{4}{3}, \frac{4}{5}, \frac{4}{6}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{5}{6}, \frac{6}{2}, \frac{6}{3}, \frac{6}{4}, \frac{6}{5}$

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outcomes in interval  $(1, 1.5)$  are  $\frac{4}{3}, \frac{5}{4}, \frac{6}{5}$

PROBABILITY IS  $\frac{3}{20}$

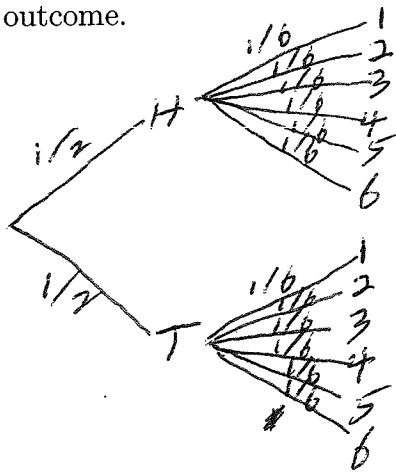
9. An experiment consists of flipping a fair coin and then rolling a standard 6-sided die.

(a) What is the sample space? Are the outcomes equally likely?

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

YES - equally likely outcomes

- (b) Make a probability tree to show all possible outcomes along with the probabilities of each outcome.



each outcome  
has probability  
 $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

- (c) What is the probability that the coin flip result is a *head* and the die result in a 5?

$$P(H5) = 1/12 \quad (\text{i.e. } \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12})$$

- (d) What is the probability that the coin flip result is a *tail* and the die result is even?

$$P(T2 \text{ or } T4 \text{ or } T6) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

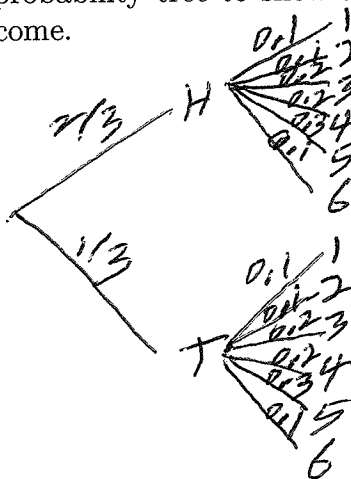
10. A magician has a coin which when flipped results in *heads*  $2/3$  of the time. He also has an unfair 6-sided die. It results in 1 (10% of the time), 2 (10% of the time), 3 (20% of the time), 4 (20% of the time), 5 (30% of the time), or 6 (10% of the time). His experiment consists of flipping the coin and then rolling the die.

- (a) What is the sample space? Are the outcomes equally likely?

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

not  
equally  
likely

- (b) Make a probability tree to show all possible outcomes along with the probabilities of each outcome.



$$\begin{aligned} P(H1) &= 2/3 (0.1) \\ P(H2) &= 2/3 (0.1) \\ P(H3) &= 2/3 (0.2) \\ P(H4) &= 2/3 (0.2) \\ P(H5) &= 2/3 (0.3) \\ P(H6) &= 2/3 (0.1) \end{aligned}$$

$$\begin{aligned} P(T1) &= 1/3 (0.1) \\ P(T2) &= 1/3 (0.1) \\ P(T3) &= 1/3 (0.2) \\ P(T4) &= 1/3 (0.2) \\ P(T5) &= 1/3 (0.3) \\ P(T6) &= 1/3 (0.1) \end{aligned}$$

- (c) What is the probability that the coin flip result is a *head* and the die result in a 5?

$$P(H5) = \frac{2}{3} \cdot (0.3) = \frac{1}{5} \text{ or } 0.2$$

- (d) What is the probability that the coin flip result is a *tail* and the die result is even?

$$\begin{aligned} P(T2 \text{ or } T4 \text{ or } T6) &= \frac{1}{3}(0.1) + \frac{1}{3}(0.2) + \frac{1}{3}(0.1) \\ &= \frac{2}{15} \end{aligned}$$

11. A purse contains three identical-looking keys, but only two of the three keys will unlock the side door of a house. Answer the following questions if two of the three keys are randomly selected.

(a) What is the sample space? Are the outcomes equally likely?

let  $G_1$  and  $G_2$  refer to the good keys  
and  $B$  refer to the bad key.

$S = \{G_1G_2, G_1B, G_2B\}$  - outcomes are  
equally likely

(b) What is the probability of selecting one key that will open the door and one key that will not?

$$\frac{2}{3}$$

(c) What is the probability of selecting both keys that will open the door?

$$\frac{1}{3}$$

12. Six cards are placed face down on a table. Four of those cards are *hearts* and two are *clubs*.

(a) If you randomly select two of these cards (without replacement), what is the probability that both of them will be *hearts*?

$$\frac{4}{6} \cdot \frac{3}{5} = \frac{2}{5}$$

(b) If you randomly select one card, replace it, mix up the 6 cards, and randomly select a 2nd card, what is the probability that both of them will be *hearts*?

$$\frac{4}{6} \cdot \frac{4}{6} = \frac{4}{9}$$

13. A standard deck of 52 cards is shuffled thoroughly and the top 3 cards are selected.

(a) What is the probability that all three cards are *hearts*?

$$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{1716}{132600}$$

(b) What is the probability that all three cards are of a black suit?

$$\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} = \frac{15600}{132600}$$

(c) What is the probability that all three cards are face cards?

$$\frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \frac{1320}{132600}$$

(d) What is the probability that all three cards are two's?

$$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{24}{132600}$$

14. One card is randomly selected from a standard deck of 52 cards and its value is recorded. It is replaced, the deck is reshuffled, a 2nd card is randomly selected, and its value is recorded. Again it is replaced, the deck is reshuffled, a 3rd card is randomly selected, and its value is recorded.

(a) What is the probability that all three cards are *hearts*?

$$\frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52} = \frac{2197}{140608}$$

(b) What is the probability that all three cards are of a black suit?

$$\frac{26}{52} \cdot \frac{26}{52} \cdot \frac{26}{52} = \frac{17576}{140608}$$

(c) What is the probability that all three cards are face cards?

$$\frac{12}{52} \cdot \frac{12}{52} \cdot \frac{12}{52} = \frac{1728}{140608}$$

(d) What is the probability that all three cards are two's?

$$\frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{64}{140608}$$