

Probability Worksheet # 2

- Evaluate the following quantities.
 - $5!$
 - $\frac{8!}{6!}$
 - $4! + 3!$
 - $(4 + 3)!$
 - $0!$
- Suppose that two fair six-sided dice (one red and one blue) are tossed. Let A denote the event that the sum is 6, let B denote the event that the sum is 7, and let C denote the event that the red die is a 4.
 - Are events A and C independent?
 - Are events B and C independent?
- Tom and Sarah each flip a coin. If they each get tails then Tom pays Sarah \$1. If they each get heads then Tom pays Sarah \$5. If their coin flips do not match then Sarah pays Tom \$2.50. Is this a fair game? If not then who can expect to benefit the most from this game and what are the expected winnings per game?
- In how many different orders can the four letters A, B, C, D be written, no letter being repeated in any one arrangement?
- How many different license plates can be made if each plate has three letters followed by 4 digits? Answer the question twice – first if repetition is allowed and then if repetition is not allowed.
- How many 4-digit positive integers are there? (No leading zeroes allowed.)
 - How many 4-digit positive integers have distinct digits?
 - How many 4-digit positive integers have only odd digits?
 - How many 4-digit positive integers are even?
 - How many 4-digit positive integers are even and have distinct digits?

7. A club has 20 members (8 men and 12 women).
- (a) How many ways can the club elect 3 of its members where one will serve as president, one as vice-president, and one as treasurer?
 - (b) How many ways can the club choose 3 of its members to serve in equal capacity on a committee?
 - (c) How many ways can the club choose 4 of its members to serve in equal capacity on a committee if the committee must consist of precisely two men and two women?
 - (d) How many ways can the club choose 4 of its members to serve in equal capacity on a committee if the committee must consist of at least two women?
8. How many distinct positive divisors do each of the following numbers have?
- (a) $2^5 \times 3^2 \times 5^4 \times 7$
 - (b) 3400
9. In a standard 52-card deck, each of the 13 card values $A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$ occurs 4 times (once in each of the 4 suits $\spadesuit, \heartsuit, \clubsuit, \diamondsuit$). A hand is a subset of the deck.
- (a) What is the total number of possible 5-card hands?
 - (b) How many 5-card hands contain a full house? A full house consists of 3 cards with one common card value and 2 cards with another common card value.
 - (c) How many 5-card hands contain a straight flush? A straight flush is 5 consecutive cards from the sequence $A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A$ all in the same suit.
 - (d) How many 5-card hands contain a flush? A flush is 5 cards all in the same suit which do not form a straight flush.
 - (e) How many 5-card hands contain a straight? A straight is 5 consecutive cards of any suit which do not form a straight flush.
 - (f) What is the probability that a 5-card hand contains a full house? a straight flush? a flush? a straight?

10. Suppose that each of 7 people are asked to think of an integer between 1 and 20 (inclusive). What is the probability that some two of them choose the same number?
11. Suppose that 23 people are chosen at random from a crowd. Show that the probability that some two of them share the same birthday (just the day, not the day and year) is greater than $1/2$. Assume no one was born on February 29th.