

MATH 103 – Arithmetic in Various Bases, Addition Algorithms

1. What are the first 15 counting numbers in each of the following bases?

(a) Base *six*

~~1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23~~

(b) Base *two* (binary)

1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011,
1100, 1101, 1110, 1111

2. Make the following conversions.

(a) Convert the base ten number 508 to base five.

$$\begin{aligned} 508_{\text{ten}} &= 4 \times 125 + 5 + 3 \\ &= 4 \times 5^3 + 0 \times 5^2 + 1 \times 5^1 + 3 \times 5^0 \\ &= 4013_{\text{five}} \end{aligned}$$

(b) Convert 372_{eight} to base ten.

$$\begin{aligned} 372_{\text{eight}} &= 3 \times 8^2 + 7 \times 8^1 + 2 \times 8^0 \\ &= 3 \times 64 + 7 \times 8 + 2 \times 1 \\ &= 192 + 56 + 2 \\ &= 250_{\text{ten}} \end{aligned}$$

(c) Convert 10101_{two} to base seven.

$$\begin{aligned} 10101_{\text{two}} &= 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 16 + 0 + 4 + 0 + 1 \\ &= 21_{\text{ten}} \\ &= 3 \times 7^1 + 0 \times 7^0 \\ &= 30_{\text{seven}} \end{aligned}$$

3. Compute the following sums using the bases shown and sketch the base pieces that represent each term in your computation. Compare this to the standard algorithm for addition.

(a) $65 + 58 = 123$

65
+ 58
123

+

+
→ □
regroup

(b) $124_{five} + 34_{five} = 213_{five}$

124
+ 34
213

□
+
□

□
→ □ □
regroup

(c) $23_{four} + 31_{four} = 120_{four}$

23
+ 31
120

+

→ □
regroup

(d) $111_{two} + 101_{two} = 1100_{two}$

111
+ 101
1100

□ □ □
+ □ □
□ □ □ □

□ □ □ □
→ □ □
regroup

↑
1 long-flat

↑
1 flat

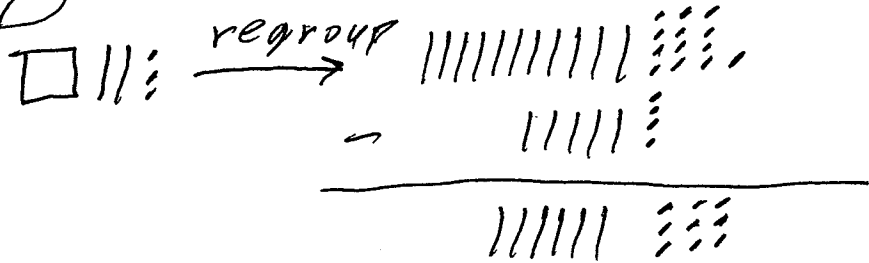
↑
0 long

↑
0 units

4. Compute the following differences using the bases shown and sketch the base pieces that represent each term in your computation. Compare this to the standard algorithm for subtraction.

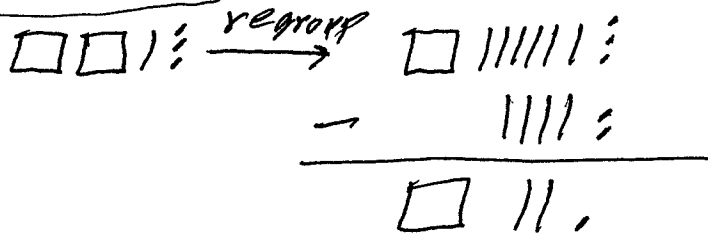
(a) $123 - 54 = 69$

$$\begin{array}{r} 0 \text{ } 1 \text{ } 2 \\ 123 \\ - 54 \\ \hline 69 \end{array}$$



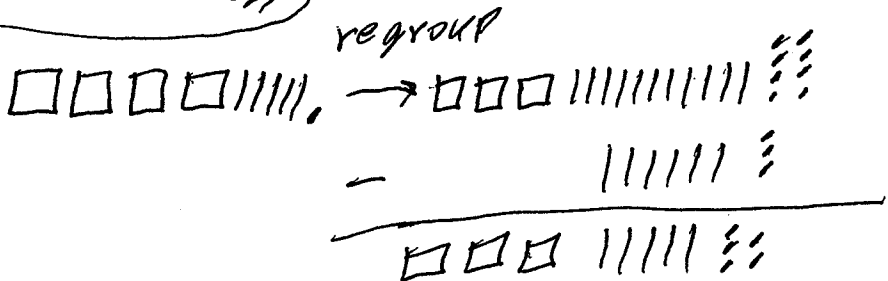
(b) $213_{\text{five}} - 42_{\text{five}} = 121_{\text{five}}$

$$\begin{array}{r} 1 \\ 213 \\ - 42 \\ \hline 121 \end{array}$$



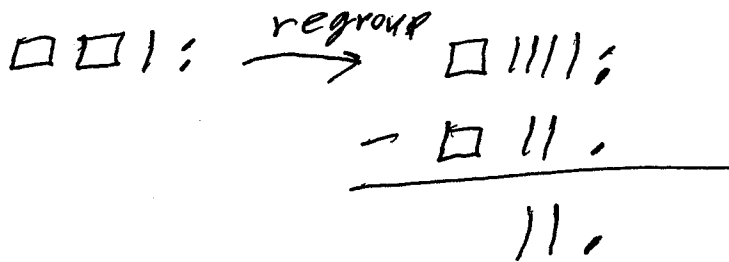
(c) $451_{\text{seven}} - 63_{\text{seven}} = 355_{\text{seven}}$

$$\begin{array}{r} 3 \text{ } 1 \text{ } 4 \\ 451 \\ - 63 \\ \hline 355 \end{array}$$



(d) $212_{\text{three}} - 121_{\text{three}} = 21_{\text{three}}$

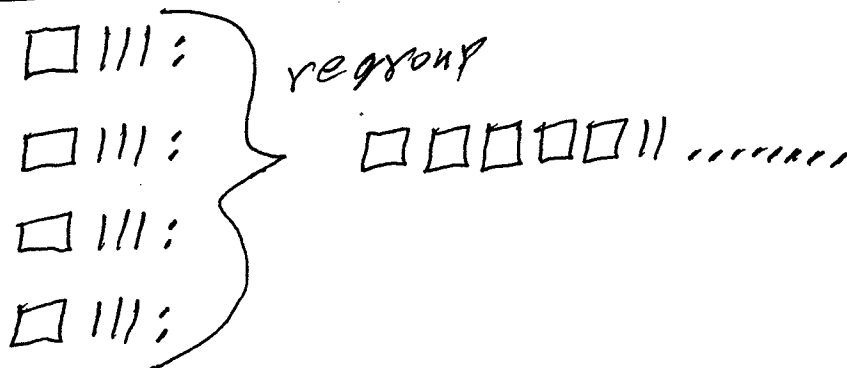
$$\begin{array}{r} 1 \\ 212 \\ - 121 \\ \hline 21 \end{array}$$



5. Products of positive integers are sometimes viewed as repeated sums. Sketch base pieces that represent the following products as repeated sums and regroup to determine that sum.

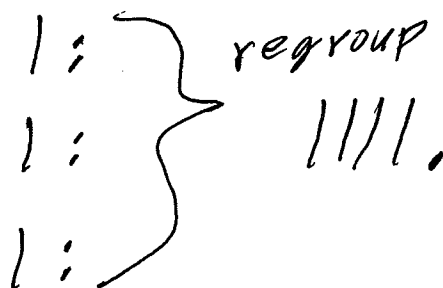
(a) $4 \times 132 = 528$

$$\begin{array}{r} 132 \\ \times 4 \\ \hline 528 \end{array}$$



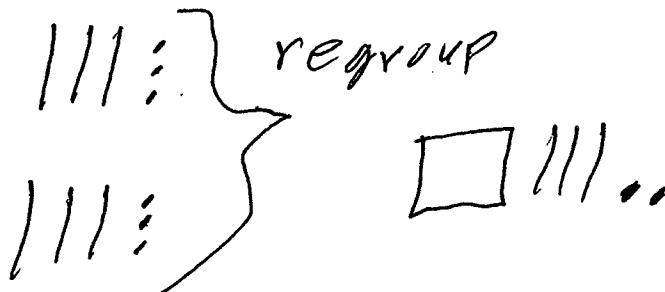
(b) $3_{\text{five}} \times 12_{\text{five}} = 41_{\text{five}}$

$$\begin{array}{r} 12 \\ \times 3 \\ \hline 41 \end{array}$$



(c) $2_{\text{four}} \times 33_{\text{four}} = 132_{\text{four}}$

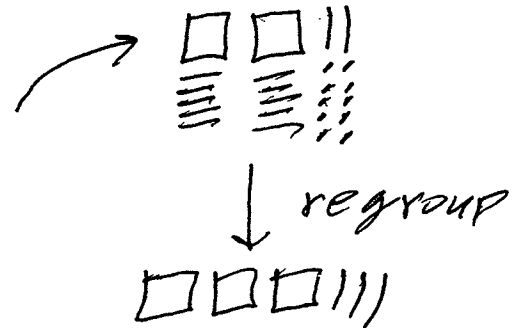
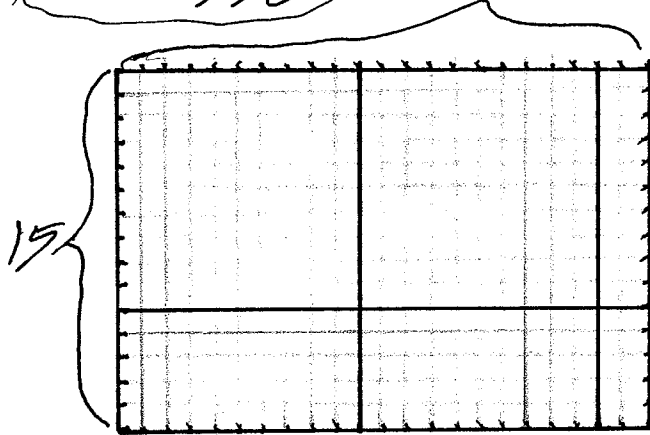
$$\begin{array}{r} 33 \\ \times 2 \\ \hline 132 \end{array}$$



6. Calculate each product in the given base by making a rectangular grid and covering it with the minimum number of pieces for that base.

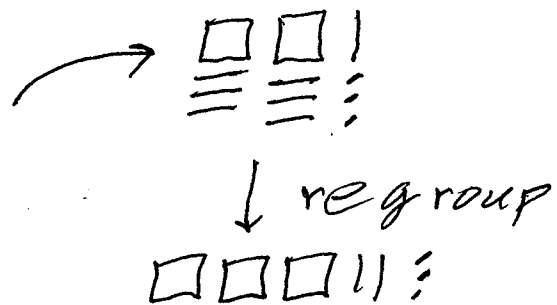
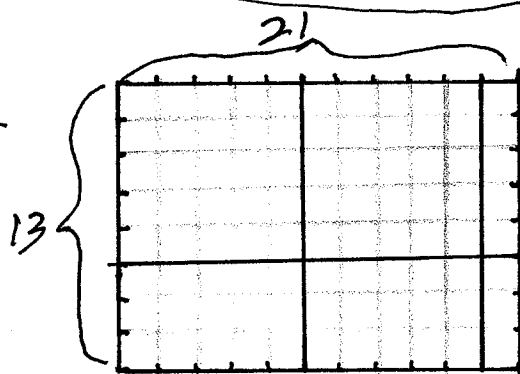
(a) $22 \times 15 = 330$

$$\begin{array}{r} 22 \\ \times 15 \\ \hline 110 \\ 22 \\ \hline 330 \end{array}$$



(b) $13_{\text{five}} \times 21_{\text{five}} = 323_{\text{five}}$

$$\begin{array}{r} 13 \\ \times 21 \\ \hline 13 \\ 31 \\ \hline 323 \end{array}$$



7. In base ten we find that multiplying by 10 is very easy. Is there an analogous computation that is easy to do in other bases?

Yes, in base b you can multiply by $10_{\text{base } b}$ by simply appending a 0 at the right side of the number.

For example, $243_{\text{five}} \times 10_{\text{five}} = 2430_{\text{five}}$

This holds for any base since

$$(d_n b^n + d_{n-1} b^{n-1} + \dots + d_2 b^2 + d_1 b + d_0)(1 \times b + 0) = d_n b^{n+1} + d_{n-1} b^n + \dots + d_2 b^3 + d_1 b^2 + d_0 b + 0$$

8. The book describes right-to-left, left-to-right, and partial sum algorithms for addition. Calculate the following sums using each of these three algorithms.

(a) $347 + 189$

$$\begin{array}{r} \overset{1}{3}47 \\ + 189 \\ \hline 536 \end{array}$$

$$\begin{array}{r} 347 \\ + 189 \\ \hline 426 \\ 53 \end{array}$$

$$\begin{array}{r} 347 \\ + 189 \\ \hline 16 \\ 12 \\ 4 \\ \hline 536 \end{array}$$

(b) $324_{\text{five}} + 241_{\text{five}}$

$$\begin{array}{r} \overset{1}{3}24_{\text{five}} \\ + 241_{\text{five}} \\ \hline 1120_{\text{five}} \end{array}$$

$$\begin{array}{r} 324_{\text{five}} \\ + 241_{\text{five}} \\ \hline 1010 \\ 12_{\text{five}} \end{array}$$

$$\begin{array}{r} 324_{\text{five}} \\ + 241_{\text{five}} \\ \hline 10 \\ 11 \\ 10 \\ \hline 1120_{\text{five}} \end{array}$$

(c) $667_{\text{eight}} + 54_{\text{eight}}$

$$\begin{array}{r} \overset{1}{6}67_{\text{eight}} \\ + 54_{\text{eight}} \\ \hline 743_{\text{eight}} \end{array}$$

$$\begin{array}{r} 667_{\text{eight}} \\ + 54_{\text{eight}} \\ \hline 633_{\text{eight}} \\ 74 \end{array}$$

$$\begin{array}{r} 667_{\text{eight}} \\ + 54_{\text{eight}} \\ \hline 13 \\ 13 \\ 6 \\ \hline 743_{\text{eight}} \end{array}$$