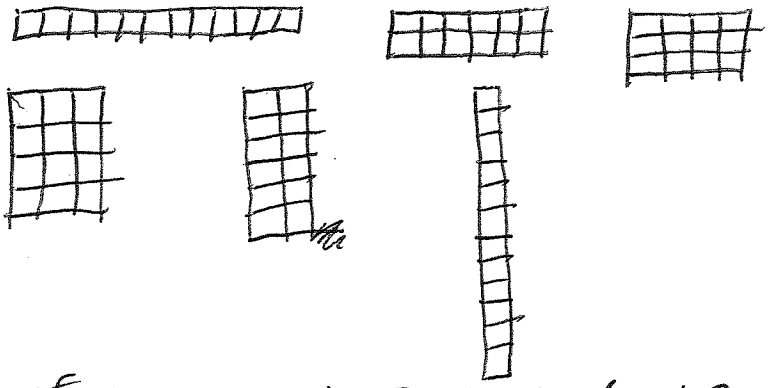
Primes, Factorization, Greatest ~~Common Factor~~, Least Common Multiple

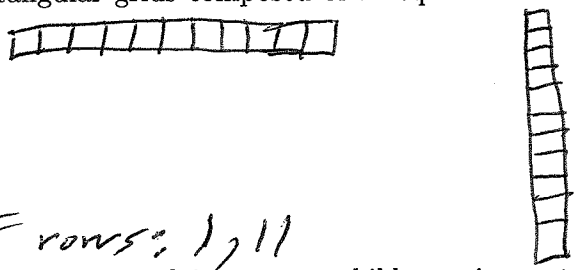
Solutions

1. Draw all rectangular grids composed of 12 squares. What is the number of rows used in each grid?



number of rows: 1, 2, 3, 4, 6, 12

2. Draw all rectangular grids composed of 11 squares. What is the number of rows used in each grid?



number of rows: 1, 11

3. What concept can you explain to young children using rectangular grids like the ones above?

Prime numbers. If a rectangular grid has more than one square, we get a prime number of squares when only two grids can be drawn - (1×n and n×1)
You can also explain multiplication, and divisibility.

4. Is 1 prime? Why or why not? Does this conflict with your explanation for how the rectangular grids are used in the first two problems?

1 is not prime. The definition of prime requires that the number is larger than 1 so that unique factorization still holds.

$n > 1$

5. List in increasing order all primes less than 100. What do we call positive integers which are not prime? What about 1 – is it prime, composite or neither?

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 are prime

4, 6, 8, 9, 10, ... are composite / 1 is neither

6. Explain how the *Prime Number Test* or the *Sieve of Eratosthenes* can help determine which numbers are prime.

→ To check if N is prime, you only need to check for divisibility by primes less than or equal to $\sqrt{N}$

see page 228 for an explanation.

7. Which of your list of primes would you have to consider as possible factors of 391 in order to determine whether 391 is a prime or composite number? Is 391 prime or composite?

since $\sqrt{391}$ is between 19 and 20, we only check divisibility by 2, 3, 5, 7, 11, 13, 17, 19

we obtain $391 = 17 \cdot 13$

so is composite.

8. Is 223 prime or composite?

since $\sqrt{223}$ is between 14 and 15, we only check divisibility by 2, 3, 5, 7, 11, 13

we obtain that 223 is prime.

9. Which one of the following numbers is prime?

(a) 209

11 19

(b) 267

3 89

(c) 317

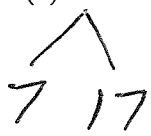
prime

(d) 341

11 31

10. Which one of the following numbers is prime?

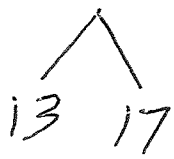
(a) 119



(b) 211

prime

(c) 221



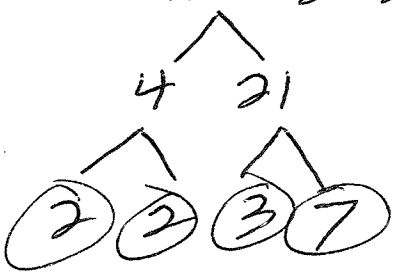
(d) 253

11. What is the *Fundamental Theorem of Arithmetic* ?

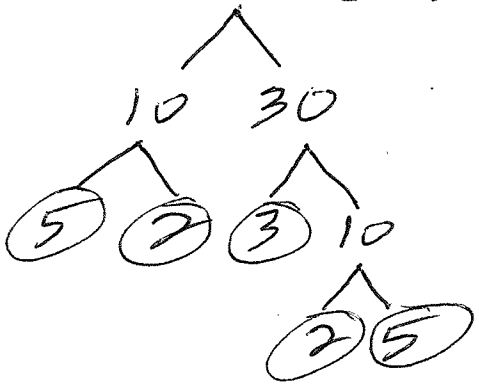
Every composite whole number can be expressed as the product of primes in exactly one way except for the order of the factors in the product.

12. Use a factor tree to determine the prime factorization of the following numbers.

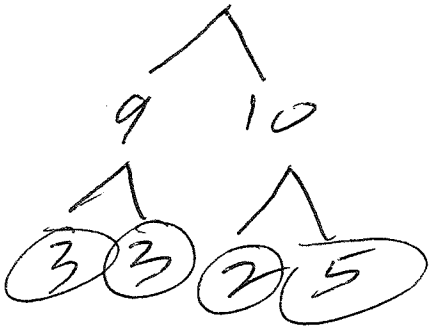
(a) 84 $= 2^2 \cdot 3 \cdot 7$



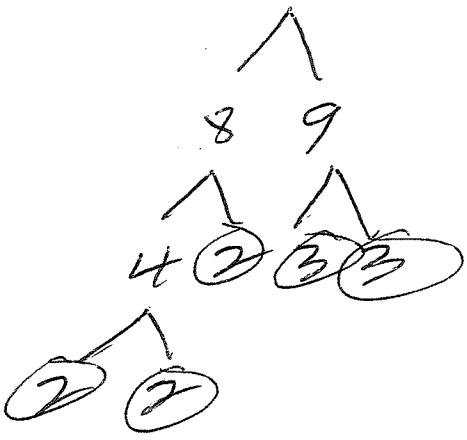
(b) 300 $= 2^2 \cdot 3 \cdot 5^2$



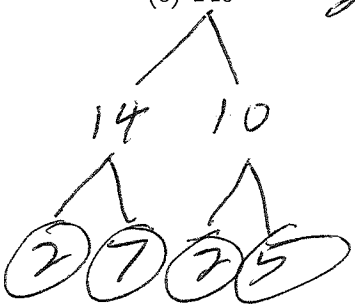
(c) 90 $= 2 \cdot 3^2 \cdot 5$



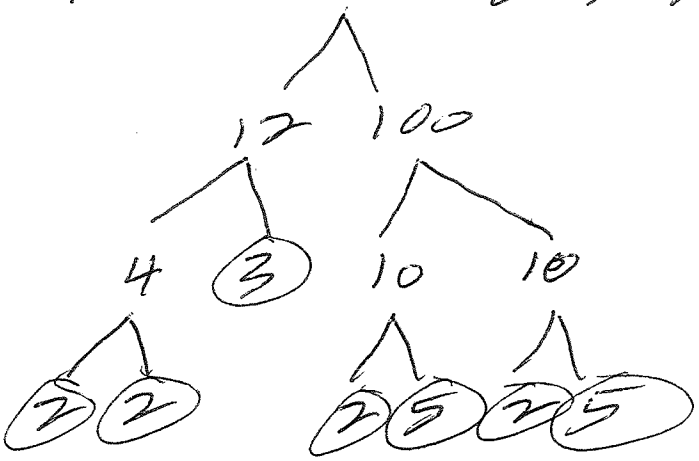
(d) 72 $= 2^3 \cdot 3^2$



(e) 140 $= 2^2 \cdot 5 \cdot 7$



(f) 1200 $= 2^4 \cdot 3 \cdot 5^2$



13. Explain how you can use rods of different lengths to convey the concept of *Least Common Multiple* to children. How can the number line be used to explain this concept?

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14. (a) List all the factors of 48.

1, 2, 3, 4, 6, 8, 12, 16, 24, 48

(b) List all the factors of 36.

1, 2, 3, 4, 6, 9, 12, 18, 36

(c) What are the common factors of 48 and 36?

1, 2, 3, 4, 6, 12

(d) What is the greatest common factor (GCF) of 48 and 36?

12

15. (a) List all the factors of 140.

1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, 140

(b) List all the factors of 150.

1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150

(c) What are the common factors of 140 and 150?

1, 2, 5, 10

(d) Determine $GCF(140, 150)$.

10

16. (a) List the beginning of the sequence of multiples of 30.

30, 60, 90, 120, ...

(b) List the beginning of the sequence of multiples of 12.

12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ...

(c) List the beginning of the sequence of common multiples of 30 and 12.

60, 120, 180, ...

(d) What is the least common multiple (LCM) of 30 and 12.

60

17. (a) List the beginning of the sequence of multiples of 55.

55, 110, 165, 220, 275, ...

(b) List the beginning of the sequence of multiples of 20.

20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, ...

(c) List the beginning of the sequence of common multiples of 55 and 20.

220, 440, 660, ...

(d) Determine $LCM(55, 20)$.

220

18. The numbers 15359 and 17041 are each prime. Determine $GCF(15359, 17041)$.

1

19. The numbers 97 and 101 are each prime. Determine $LCM(97, 101)$.

$97 \cdot 101 = 9797$

20. Todd and Nancy each have one bookcase. Each shelf in Todd's bookcase contains the same number of books as any other shelf in his bookcase. Each shelf in Nancy's bookcase contains the same number of books as any other shelf in her bookcase. Furthermore, Todd's bookcase has the same number of shelves as Nancy's bookcase. If Todd has 252 books and Nancy has 330 books, then what is the largest number of shelves they could possibly have in each of their bookcases?

Let N be the number of shelves in each bookcase. Then $N \mid 252$ and $N \mid 330$. We need $GCF(252, 330)$.

$GCF(252, 330) = GCF(2^2 \cdot 3^2 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 11)$

$= 2 \cdot 3 = 6$

21. Midas has 120 gold coins and 96 silver coins. He wants to place his gold coins and his silver coins in stacks so that there are the same number of coins in each stack. What is the greatest number of coins that he can place in each stack?

Let N be the number of gold coins in each gold stack and the number of silver coins in each silver stack. Then $N \mid 120$ and $N \mid 96$. We need $GCF(120, 96)$.

$GCF(120, 96) = GCF(2^3 \cdot 3 \cdot 5, 2^5 \cdot 3) = 2^3 \cdot 3 = 24$

22. Use any method to evaluate the following.

(a) $GCF(12, 30) = 6$

(b) $LCM(12, 30) = 60$

(c) $GCF(45, 30) = 15$

(d) $LCM(45, 30) = 90$

(e) $GCF(2016, 548800)$ Hint: $2016 = 2^5 \cdot 3^2 \cdot 7$ and $548800 = 2^6 \cdot 5^2 \cdot 7^3$

$= 2^5 \cdot 7 = 224$

(f) $LCM(2016, 548800)$

$= 2^6 \cdot 3^2 \cdot 5^2 \cdot 7^3$

(g) $GCF(121500, 1148175)$ Hint: $121500 = 2^2 \cdot 3^5 \cdot 5^3$ and $1148175 = 3^8 \cdot 5^2 \cdot 7$

$= 3^5 \cdot 5^2$

(h) $LCM(121500, 1148175)$

$= 2^2 \cdot 3^8 \cdot 5^3 \cdot 7$

(i) $GCF\left(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2, 2^2 \cdot 3 \cdot 5^2\right) = 2 \cdot 3 = 6$

(j) $LCM\left(2^3 \cdot 3 \cdot 5, 2 \cdot 3^2, 2^2 \cdot 3 \cdot 5^2\right) = 2^3 \cdot 3^2 \cdot 5^2 = 1800$

23. Two positive integers a and b satisfy $GCF(a, b) = 150$ and $a \cdot b = 1350000$. Compute the value of $LCM(a, b)$.

$$LCM(a,b) = \frac{a \cdot b}{GCF(a,b)} = \frac{1350000}{150} = 9000$$