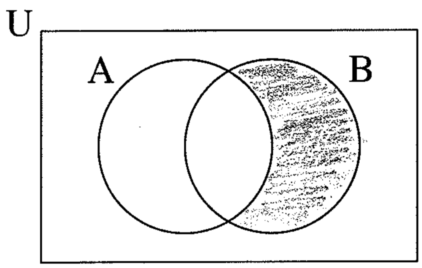
SOLUTIONS

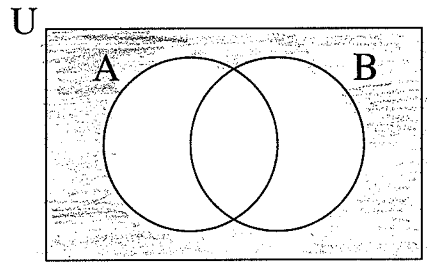
Sets and Venn Diagrams

1. Use set notation to identify the shaded region shown in each Venn diagram.



$B \cap A'$

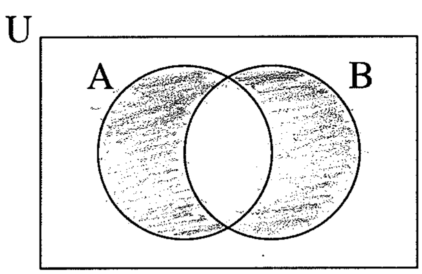
$= (B' \cup A)'$



$(A \cup B)'$

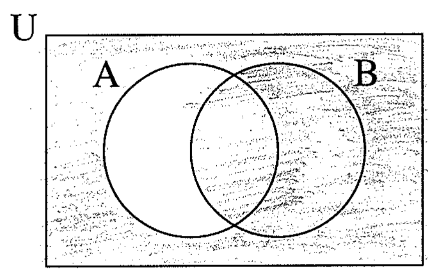
$= A' \cap B'$

some of many acceptable answers shown



$(A \cap B') \cup (B \cap A')$

$= (A \cup B) \cap (A \cap B)'$

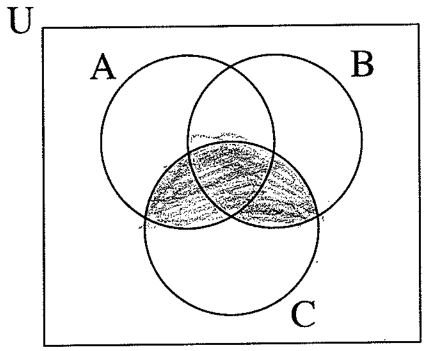


$A' \cup B$

$= (A \cap B')'$

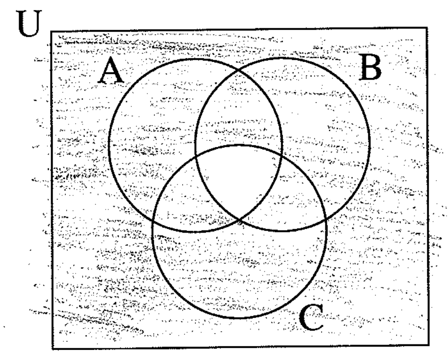
$= B \cup (A \cup B)'$

$= A' \cup (A \cap B)$



$C \cap (A \cup B)$

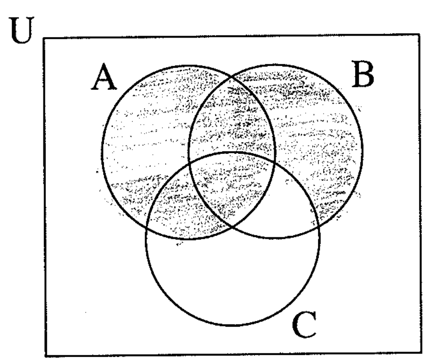
$= (C \cap A) \cup (C \cap B)$



$(A \cap B \cap C)'$

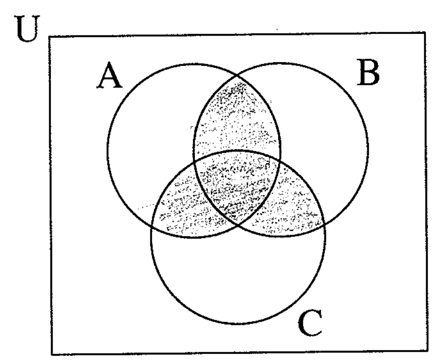
$= A' \cup B' \cup C'$

some of many acceptable answers shown



$A \cup (B \cap C')$

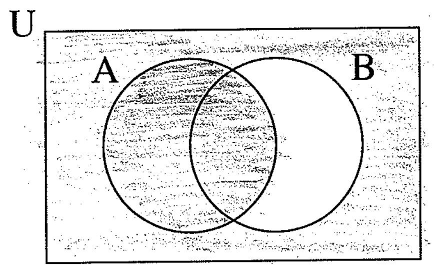
$= (A \cup B) \cap (A \cap C')$



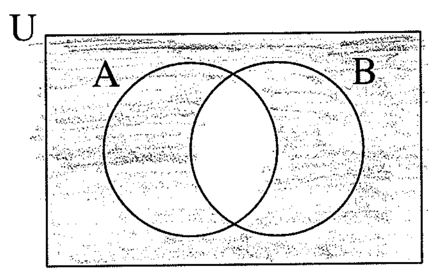
$(A \cap B) \cup (A \cap C) \cup (B \cap C)$

2. Using a 2-circle Venn diagram with universal set U, shade in the portion which represents each of the following sets.

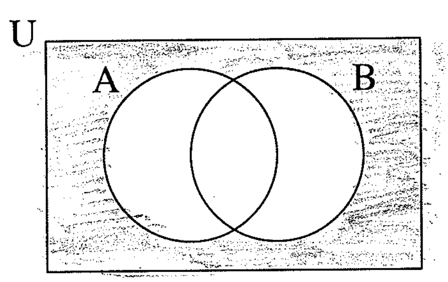
(a) $A \cup B'$



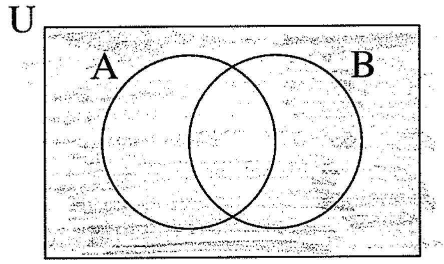
(b) $(A \cap B)'$



(c) $A' \cap B'$

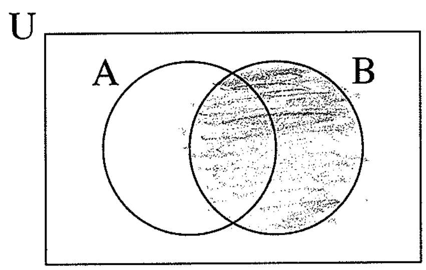


(d) $A \cup A'$



$A \cup A' = U$

(e) $(A \cup A') \cap B$

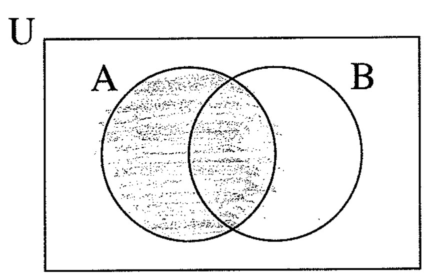


$(A \cup A') \cap B =$

$U \cap B =$

B

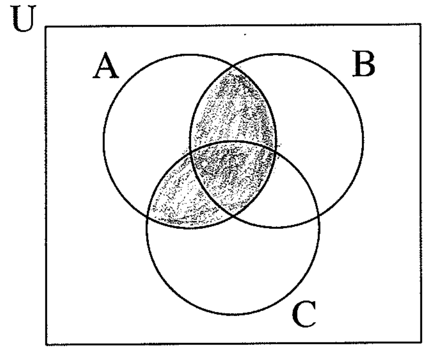
(f) $A \cup \emptyset$



$A \cup \emptyset = A$

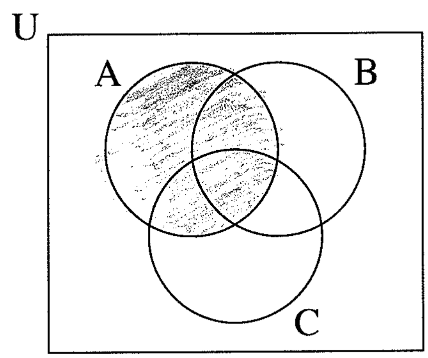
3. Using a 3-circle Venn diagram with universal set U, shade in the portion which represents each of the following sets.

(a) $A \cap (B \cup C)$



(b) $A \cap (A \cup B \cup C)$

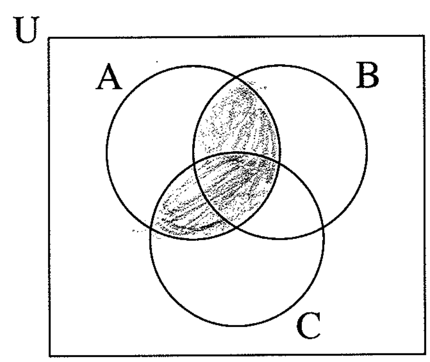
$= A$



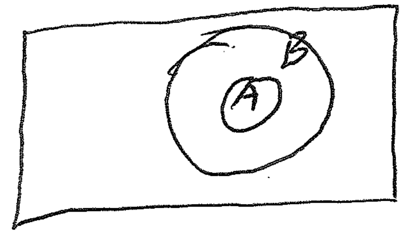
(c) $(A \cap B) \cup (A \cap C)$

note from part (a) that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



4. If $A \subseteq B$, then find a simpler way to express the following sets.



(a) $A \cap B$ $= A$

(b) $A \cup B$ $= B$

(c) $A \cap B'$ $= \emptyset$

(d) $A \cup \emptyset$ $= A$

this is true even if

$A \not\subseteq B$

5. Rewrite each of the following sets by explicitly listing the elements of the set. How many elements are in each set?

(a) $A = \{k \mid k \text{ is a perfect square which is } \leq 30\}$

$A = \{0, 1, 4, 9, 16, 25\}$

$|A| = 6$

(b) $A = \{2i - j \mid i \in \{2, 3, 4\} \text{ and } j \in \{1, 2, 3\}\}$

$2(2) - 1 = 3,\ 2(2) - 2 = 2,\ 2(2) - 3 = 1$

$2(3) - 1 = 5,\ 2(3) - 2 = 4,\ 2(3) - 3 = 3$

$2(4) - 1 = 7,\ 2(4) - 2 = 6,\ 2(4) - 3 = 5$

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$|A| = 7$

6. Circle **T** for true or **F** for false for each of following statements.

(a) **T** or **F**: $\{1\} \in \{1,2,3\}$ but $1 \in \{1,2,3\}$ and $\{1\} \subseteq \{1,2,3\}$

(b) **T** or **F**: $2 \in \{1,2,3\}$

(c) **T** or **F**: $3 \subseteq \{1,2,3\}$ but $3 \in \{1,2,3\}$ and $\{3\} \subseteq \{1,2,3\}$

(d) **T** or **F**: $\{1,2\} \subseteq \{1,2\}$

(e) **T** or **F**: $\{1,2\} \subset \{1,2\}$ (not a proper subset)

(f) **T** or **F**: $\{1,2\} \subset \{1,2,3\}$

(g) **T** or **F**: $\{1,2,3\} \subset \{1,2\}$

(h) **T** or **F**: $\{1,2\} \not\subseteq \{1,2,3\}$

(i) **T** or **F**: $0 \in \emptyset$ empty set $\emptyset$ has no elements

(j) **T** or **F**: $\emptyset \subseteq \{1,2\}$ empty set is a subset of every set

(k) **T** or **F**: $\{4\} \notin \{1,2,3\}$

7. In this problem, the universal set is the set of all humans,

$$B = \{x|x \text{ is a college basketball player}\}$$

and

$$S = \{x|x \text{ is a college student whose height is more than 200 cm tall}\}$$

Which of the following is the set of all college basketball players whose height is less than or equal to 200 cm?

(a) $B \cap S$

(b) $B \cup S$

(c) $B \cap S'$

(d) S'

8. What are De Morgan's laws?

For 2 sets,

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

For 3 sets,

(iii) $(A \cup B \cup C)' = A' \cap B' \cap C'$

(iv) $(A \cap B \cap C)' = A' \cup B' \cup C'$

9. If $U = \{a, b, c, d, e\}$ is the universal set and A and B are sets satisfying $A' \cup B' = \{a, c, e\}$, what is the set $A \cap B$?

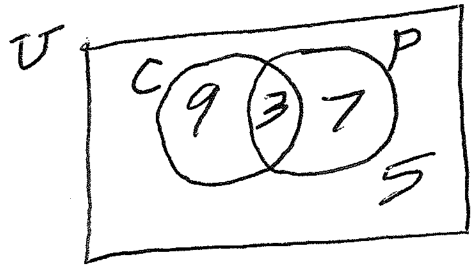
~~For~~ From De Morgan's Law (ii) we have

$$(A \cap B)' = A' \cup B'$$

$$(A \cap B)' = \{a, c, e\}$$

so $A \cap B = \{b, d\}$

10. In a group of students, 12 are taking chemistry, 10 are taking physics, 3 are taking both and 5 are taking neither. How many students are in the group?



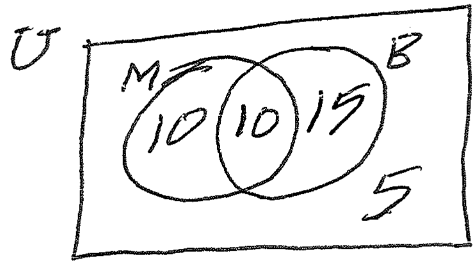
$9 + 3 + 7 + 5 = 24$

or from inclusion-exclusion principle,

$$|C \cup P| = |C| + |P| - |C \cap P|$$
$$= 12 + 10 - 3$$
$$= 19$$ who are taking chem. or physics

add 5 who take neither to get 24

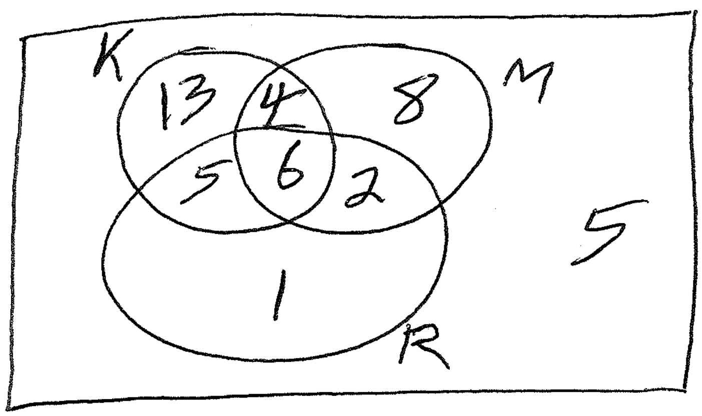
11. In a fraternity with 40 members, 20 take mathematics, 10 take both mathematics and biology, and 5 take neither mathematics nor biology. How many take biology but not mathematics?



$10 + 10 + 5 + \boxed{} = 40$

15

12. A coach offered to buy hot dogs for the players on the team. Of the 44 players, 28 wanted ketchup, 20 wanted mustard, 14 wanted relish, 10 wanted ketchup and mustard, 11 wanted ketchup and relish, 8 wanted mustard and relish and 6 wanted all three condiments. How many players wanted



(a) Ketchup only? 13

(b) Mustard but not relish? $8 + 4 = 12$

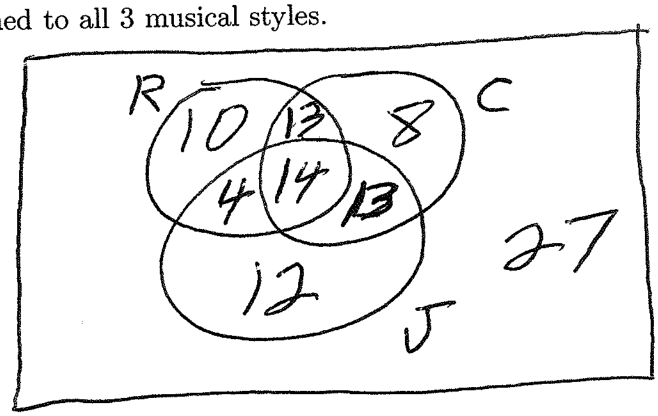
(c) Relish but not mustard? $1 + 5 = 6$

(d) Ketchup and mustard but not relish? 4

(e) Relish and mustard but not ketchup? 2

(f) None of the three condiments? 5

13. A survey of 101 college students was taken to determine if they preferred rock, classical, or jazz music. 41 students listened to rock, 48 to classical, and 43 to jazz. Furthermore 18 students listened to rock and jazz, 27 to rock and classical, and 27 to classical and jazz. Finally, 14 students listened to all 3 musical styles.



(a) How many listened only to rock music? 10

(b) How many listened to classical and jazz, but not rock? 13

(c) How many listened to classical or jazz, but not rock?

$8 + 13 + 12 = 33$

(d) How many listened to music in exactly one of the musical styles?

$$10+8+12=30$$

(e) How many listened to music in exactly two of the musical styles?

$$13+13+4=30$$

(f) How many did not listen to any of the musical styles?

$$27$$

14. A survey asked 100 people if they like Math, Science or History. Everyone answered that they like at least one of these subjects – 56 like Math, 43 like Science, 35 like History, 18 like Math and Science, 10 like Science and History, 12 like Math and History. How many like all three subjects?

From inclusion-exclusion principle,

$$|M \cup S \cup H| = |M| + |S| + |H| - |M \cap S| - |M \cap H| - |S \cap H| + |M \cap S \cap H|$$

$$100 = 56+43+35-18-12-10+|M \cap S \cap H|$$

$$100 = 94 + |M \cap S \cap H|$$

$$|M \cap S \cap H| = 6$$

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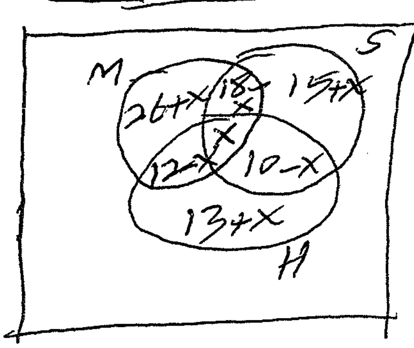
another approach



$$\begin{array}{r} 26+x \\ +\ 18-x \\ +\ 15+x \\ +\ 12-x \\ +\ x \\ +\ 10-x \\ +\ 13+x \\ \hline 94+x \end{array}$$

$$94+x=100$$

$$x=6$$