High School Math Questions

1. The total profit for a company in February was 40% higher than it was in January. The total profit for the two months was \$72000. What was the company's profit in February?

Let J be the January profit.

$J + 1.4J = 72000$

$2.4J = 72000$

$J = \frac{72000}{2.4} = 30000$

→ Feb. profit = $1.4(30000) = \$42{,}000$

2. On Friday, John went to the store and found that a particular shirt and tie had a combined cost of \$50. He had no money with him at the time, but returned the following day to purchase the two items. When he returned, he found that the cost of the shirt had been decreased by 10%, and the cost of the tie had been increased by 20%. Due to these changes, the shirt and the tie now had a combined cost of \$51.75. What was the cost of the shirt when he first went into the store on Friday?

$S + T = 50 \Rightarrow T = 50 - S$

$0.95S + 1.2T = 51.75$

$0.95S + 1.2(50 - S) = 51.75$

→ $0.95S + 60 - 1.25S = 51.75$

$8.25 = 0.25S$

$S = \$27.50$

3. Without using a calculator, simplify the quantity $\left|\frac{\pi - 3}{7}\right| + \left|\frac{\pi - 5}{7}\right|$

$\frac{\pi - 3}{7} + \left(-\frac{\pi - 5}{7}\right) = \frac{\pi - 3}{7} + \frac{-\pi + 5}{7} = \frac{2}{7}$

4. A circle has center $C(1, 3)$ and passes through the point $P(5, 6)$

(a) Find the radius of this circle.

radius = distance from C to P

$= \sqrt{(5-1)^2 + (6-3)^2} = \sqrt{25} = 5$

(b) Find an equation for this circle.

$(x-1)^2 + (y-3)^2 = 5^2$

(c) Is the point $(3, 7.5)$ inside, outside, or on this circle?

distance to center is

$\sqrt{(3-1)^2 + (7.5-3)^2} = \sqrt{4 + 20.25} = \sqrt{24.25} < 5$

inside circle

5. The number of bacteria in a certain culture at time t (in hours) is given by the formula $Q(t) = 2000\left(3^t\right)$.

(a) What is the number of bacteria at time $t = 3$?

$Q(3) = 2000(3^3) = 54000$

(b) How long does it take until the number of bacteria is equal to 18000 ?

$18000 = 2000(3^t)$

$9 = 3^t$

$t = 2$ hours

6. Suppose that $f(x) = x^2 - 4$.

(a) Find $f(-1)$.

$f(-1) = (-1)^2 - 4 = -3$

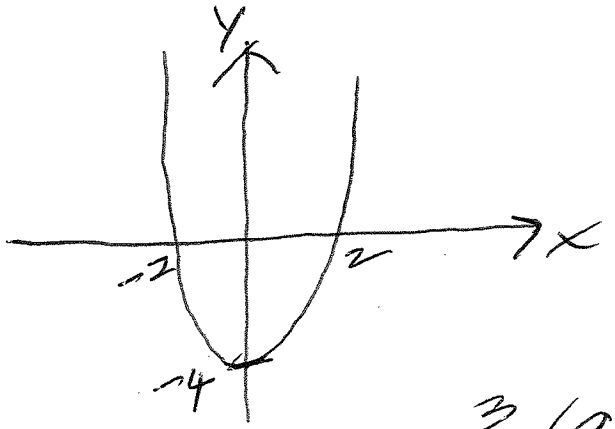
(b) Find the domain of f.

all real numbers

(c) Find the range of f.

$[-4, \infty)$

(d) Carefully sketch the graph of f, being sure to include all x-intercepts and y-intercepts.



7. Simplify the expression $2x^3(-3x^4)^2$.

$= 2x^3 \cdot (9x^8)$

$= 18x^{11}$

8. Rewrite the expression $\dfrac{(10x^{-3})^2}{20x^2}$ with positive exponents and simplify.

$\dfrac{\left(\frac{10}{x^3}\right)^2}{20x^2} = \dfrac{100/x^6}{20x^2} = \dfrac{100}{20x^8} = \dfrac{5}{x^8}$

9. The polynomial $(x+4)^2 - 4(3x+5)$ is equal to

$x^2 + 8x + 16 - 12x - 20$

$x^2 - 4x - 4$

10. The polynomial $(x+3)(x^2 - 3x + 5)$ is equal to

$x^3 - 3x^2 + 5x + 3x^2 - 9x + 15$

$x^3 - 4x + 15$

11. Factor the polynomial $2x^2 - x - 15$.

$(2x+5)(x-3)$

12. Factor the polynomial $x^3 - 3x^2 + 2x - 6$.

$x^2(x-3) + 2(x-3)$

$(x^2+2)(x-3)$

13. Simplify the expression $\dfrac{2x^2 - 200}{x - 10}$.

$= \dfrac{2(x^2-100)}{x-10}$

$= \dfrac{2(x-10)(x+10)}{x-10}$

$= 2(x+10)$ or $2x+20$

14. Simplify the expression $\dfrac{8x}{2x+3} - 4$.

$\dfrac{8x}{2x+3} - \dfrac{4(2x+3)}{2x+3} = \dfrac{8x - 8x - 12}{2x+3} = \dfrac{-12}{2x+3}$

15. Find the distance between the points $(1, 4)$ and $(4, 14)$ in the Cartesian plane.

$$D = \sqrt{(4-1)^2+(14-4)^2}$$
$$= \sqrt{3^2+10^2}$$
$$= \sqrt{109}$$

16. Find all solutions to the equation $\frac{6}{x-2} = \frac{7}{3x+5}$.

$$6(3x+5) = 7(x-2)$$
$$18x+30 = 7x-14$$
$$11x = -44$$
$$x = -4$$

17. Find all solutions to the equation $x^2 = 2x + 2$.

$$x^2-2x-2 = 0$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{12}}{2}$$
$$= \frac{2 \pm 2\sqrt{3}}{2}$$
$$= 1 \pm \sqrt{3}$$

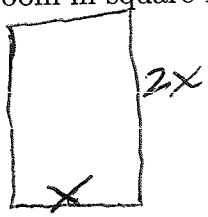
18. Solve the inequality $|7 - 2x| \leq 20$.

$$-20 \leq 7-2x \leq 20$$
$$-27 \leq -2x \leq 13$$
$$\frac{27}{2} \geq x \geq \frac{-13}{2}$$
$$\left[\frac{-13}{2}, \frac{27}{2}\right]$$

19. Solve the inequality $(x+1)^2 < 9$.

$$\sqrt{(x+1)^2} < \sqrt{9}$$
$$|x+1| < 3$$
$$-3 < x+1 < 3$$
$$-4 < x < 2 \quad \text{or} \quad (-4, 2)$$

20. A room is twice as long as it is wide, and its perimeter is 90 meters. What is the area of the room in square meters?



$90 = x + 2x + x + 2x$

$90 = 6x \Rightarrow x = 15$

AREA $= x \cdot 2x = 15 \cdot 30 =$ **450 m²**

21. Find the equation for the line which goes through the points $(2, 6)$ and $(-2, 14)$.

slope $m = (14-6)/(-2-2) = -2$

$y - 6 = -2(x-2) \Rightarrow$ **$y = -2x + 10$**

22. Find the equation of the line which contains the point $(6, 5)$ and is parallel to the line $y = 3x + 2$.

$y - 5 = 3(x-6)$

$y = 3x - 13$

same slope

23. Find the equation of the line which contains the point $(2, 3)$ and is perpendicular to the line $y = 0.5x + 5$.

$y - 3 = -2(x-2)$

$y = -2x + 7$

slope is $\frac{-1}{0.5} = -2$

24. Find the domain of the function $f(x) = \ln(16 - 2x)$.

$16 - 2x > 0$

$2x < 16$

$x < 8$ or **$(-\infty, 8)$**

25. Find the domain of the function $f(x) = \dfrac{4x}{\sqrt{6-2x}}$.

$6 - 2x > 0$

$6 > 2x$

$3 > x$ or **$(-\infty, 3)$**

26. How many x-intercepts are on the graph of $f(x) = (x^3 - 4x)(x^2 + 9)$.

$f(x) = x(x^2 - 4)(x^2 + 9)$

$f(x) = x(x-2)(x+2)(x^2+9)$

0, 2, −2 → 3 x-intercepts

27. What is the remainder when $5x^3 + 30x - 40$ is divided by $x + 2$?

$$\begin{array}{r} 5x^2 - 10x + 50 \\ x+2 \overline{)\,5x^3 + 0x^2 + 30x - 40} \\ 5x^3 + 10x^2 \\ \hline -10x^2 + 30x - 40 \\ -10x^2 - 20x \\ \hline 50x - 40 \\ 50x + 100 \\ \hline -140 \end{array}$$

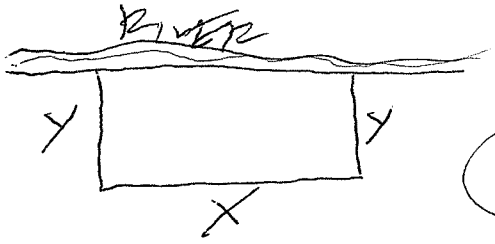
-140

28. Through polynomial division, one finds that $\dfrac{x^4+1}{x^2+1} =$

$$\begin{array}{r} x^2 - 1 \\ x^2+1 \overline{)\,x^4 + 0x^3 + 0x^2 + 0x + 1} \\ x^4 \quad + x^2 \\ \hline -x^2 + 1 \\ -x^2 - 1 \\ \hline 2 \end{array}$$

$x^2 - 1 + \dfrac{2}{x^2+1}$

29. Patty has 6000 feet of fencing available to enclose a rectangular field. One side of the field lies along a river, so only three sides require fencing. Which one of following represents the area A of the rectangular field as a function of x, where x is the length of the side parallel to the river?



$x + 2y = 6000$

$y = \dfrac{6000 - x}{2}$

$A = x \cdot y = x\left(\dfrac{6000 - x}{2}\right)$

30. Find the standard form of the quadratic function which has its vertex at $(-4, 8)$ and passes through the point $(0, 40)$.

$y = A(x - (-4))^2 + 8$

$y = A(x+4)^2 + 8$

$40 = A(0+4)^2 + 8 \Rightarrow A = 2$

$y = 2(x+4)^2 + 8$

31. Find the vertex of the parabola given by $f(x) = x^2 + 60x + 800$.

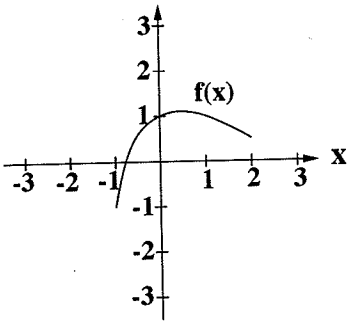
$y = x^2 + 60x + 800$

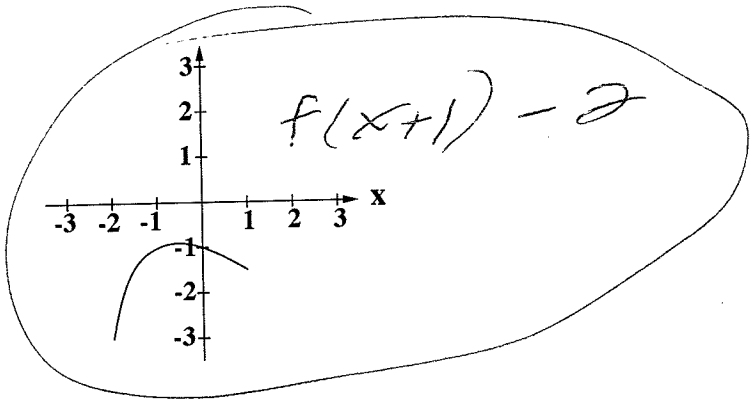
$= x^2 + 60x + 900 - 100$

$= (x+30)^2 - 100$

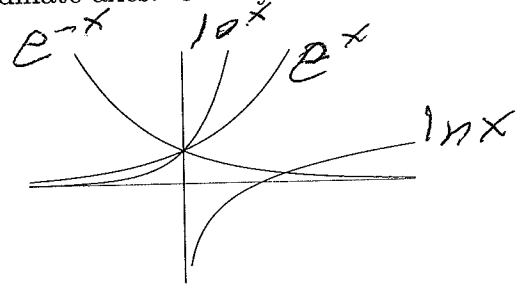
vertex at $(-30, -100)$

32. The graph of $f(x)$ is shown below. Sketch the graph of $f(x+1)-2$.





33. The graphs of the functions $y = e^x$, $y = e^{-x}$, $y = \ln(x)$, and $y = 10^x$ are shown on the same coordinate axes. Clearly indicate which graph represents each function.



34. This morning, Lois has invested \$100 in an account which earns 7.5% interest compounded annually. How much money will she have in her account 5 years from now?

$A = 100(1.075)^5 \approx \$143.56$

35. This morning, Terry has invested \$400 in an account which earns 18% interest compounded continuously. How many years will it take for the balance in his account to reach \$7000 ?

$A = 400e^{0.18t}$

$7000 = 400e^{0.18t}$

$17.5 = e^{0.18t}$

$\ln(17.5) = 0.18t$

$t = \ln(17.5)/0.18$

$t \approx 15.9$ years

36. A bank has advertised that their customers can triple their investments in only 8 years! What is the interest rate used by this bank if interest is compounded continuously?

$A = A_0 e^{rt}$

$3A_0 = A_0 e^{r \cdot 8}$

$3 = e^{8r}$

$\ln 3 = 8r$

$r = \ln 3 / 8$

$r \approx 0.137$

13.7%

37. Simplify the following quantities as much as possible.

(i) $2\ln(e^{10000})$ $= 2 \cdot 10000 = 20{,}000$

(ii) $\log_5(125)$ $= \log_5(5^3) = 3$

(iii) $\log_4(\sqrt[3]{4})$ $= \log_4(4^{1/3}) = 1/3$

38. Solve for x in each of the following equations.

(i) $\log_3(x) = 6$ $\quad x = 3^6 = 729$

(ii) $\ln(x) + \ln(x-2) = \ln(x^2+x-12)$

$\ln(x \cdot (x-2)) = \ln(x^2+x-12)$

$x^2 - 2x = x^2 + x - 12$

$12 = 3x$

$x = 4$

(iii) $27^x = 9^{x+2}$

$(3^3)^x = (3^2)^{x+2}$

$3^{3x} = 3^{2x+4}$

$3x = 2x + 4$

$x = 4$

39. Rewrite $\ln(a) + 2\ln(b) - 3\ln(c)$ as the logarithm of a single quantity.

$\ln(a) + \ln(b^2) - \ln(c^3)$

$= \ln\left(\frac{ab^2}{c^3}\right)$

40. Find the value of $\log_2(50)$ accurate to two decimal places.

$= \frac{\ln 50}{\ln 2} \approx 5.64$