1. Suppose $p(n)$ represents some population n years from now, and that this population is modeled by the following discrete dynamical system.

$$p(n) = p(n-1) + 35$$
$$p(0) = 30$$

Which one of the following statements follows from this model?

(a) The population will increase by 35 people per year.

(b) The population will increase by 30 people per year.

(c) The population will increase by 35% per year.

(d) The population will increase by 30% per year.

2. Suppose $a(n)$ represents the number of milligrams of some drug in the bloodstream n hours from now, and that the amount of this drug in the bloodstream is modeled by the following discrete dynamical system.

$$a(n) = 0.3a(n-1)$$
$$a(0) = 50$$

Which one of the following statements follows from this model?

(a) The amount of drug in the bloodstream will increase by 30% per hour.

(b) The amount of drug in the bloodstream will increase by 50% per hour.

(c) The amount of drug in the bloodstream will increase by 70% per hour.

(d) The amount of drug in the bloodstream will decrease by 30% per hour.

(e) The amount of drug in the bloodstream will decrease by 50% per hour.

(f) The amount of drug in the bloodstream will decrease by 70% per hour.

3. Consider the following discrete dynamical system.

$$a(n) = 0.9a(n-1) + 30$$

If $a(0) = 1$, then what is the value of $a(10)$?

Enter the system on your calculator to obtain $a(10) \approx 195.75$.

4. There are 2 drugs, U and V. Let $u(n)$ and $v(n)$ represent the number of milligrams of each drug in the body at the beginning of day n. The body converts 5% of U into V each day and converts 20% of V into U each day. Assume that 200 mg of U and 600 mg of V are consumed each day, and that the body eliminates 100 mg of U and 50 mg of V each day. Develop a discrete dynamical system (without initial values) to represent $u(n)$ and $v(n)$.

   u(n)=u(n-1) - 0.05u(n-1) + 0.2v(n-1) + 200 - 100

   v(n)=v(n-1) + 0.05u(n-1) - 0.2v(n-1) + 600 - 50

   or in simplified form

   u(n)=0.95u(n-1) + 0.2v(n-1) + 100

   v(n)=0.05u(n-1) + 0.8v(n-1) + 550

5. For which one of the following discrete dynamical systems is $u(n)$ oscillating toward its equilibrium value and getting 10% closer each day? (Assuming n is measured in days)

   (a) $u(n) = 0.1u(n-1) - 0.9$

   (b) $u(n) = -0.1u(n-1) + 0.9$

   (c) $u(n) = 0.9u(n-1) - 0.9$

   (d) $u(n) = -0.9u(n-1) + 0.1$

   (e) $u(n) = 1.1u(n-1) - 0.1$

   (f) $u(n) = -1.1u(n-1) + 0.1$

6. We are given the following discrete dynamical system.

$$u(n) = 0.9u(n-1) + 0.2v(n-1) + 600$$
$$v(n) = 0.1u(n-1) + 0.8v(n-1) + 400$$

   Although the function $v(n)$ is not linear, as n increases the function starts to look like a line with slope $333.\overline{3}$.

   Reasoning: By picking a starting value $v(0)$ and computing $v(n)-v(n-1)$ using a lot of different values for n, we obtain numerical evidence which suggests that $\lim_{n\to\infty}(v(n) - v(n-1)) = 333.\overline{3}$.

7. Technicium-99m is the most widely used radioisotope in diagnostic nuclear medicine. It is known that 11% of the substance is lost each hour through radioactive decay. How long does it take until only 35% of the substance remains? You may give your answer to the nearest hour.

$$u(n) = u_0(0.89)^n$$
$$0.35u_0 = u_0(0.89)^n$$
$$0.35 = (0.89)^n$$
$$\ln(0.35) = n\ln(0.89)$$
$$n = \frac{\ln(0.35)}{\ln(0.89)} \approx 9 \text{ hours}$$

8. Find an explicit formula for an expression which satisfies each of the following discrete dynamical systems given that $u(0) = 90$ in each case.

   (a) If $u(n) = u(n-1) + 10$, then $\boxed{u(n) = 10n + 90}$

   (b) If $u(n) = 0.8u(n-1)$, then $\boxed{u(n) = 90(0.8)^n}$

   (c) If $u(n) = 0.8u(n-1) + 10$, then $\boxed{u(n) = 40(0.8)^n + 50}$

9. Suppose you borrow $\$20,000$ at a 4.8% annual interest rate compounded monthly to be paid back in monthly payments of \$700.

   (a) Write down a discrete dynamical system with initial condition to represent the balance owed on the loan after making n monthly payments.

   Since $0.048/12 = 0.004$, we obtain

   $$u(n) = 1.004u(n-1) - 700$$
   $$u(0) = 20000$$

   (b) To fully pay off this loan, the first $\boxed{30}$ monthly payments will each be for \$700, but the next and final payment will be for $\boxed{\$280.93}$.