

Name _____

- You must show sufficient work to justify your answers.

#1 (20 points) _____

#2 (10 points) _____

#3 (10 points) _____

#4 (10 points) _____

#5 (10 points) _____

#6 (10 points) _____

#7 (15 points) _____

#8 (15 points) _____

Total (100 points) _____

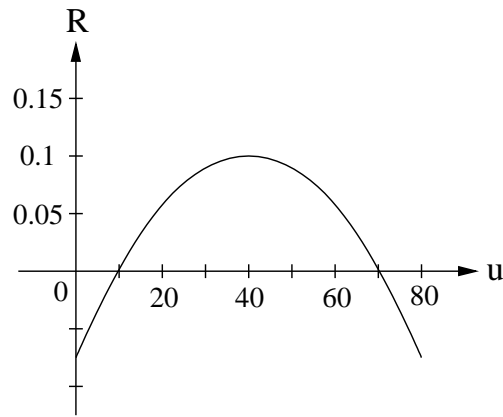
Test #1 _____ Test #2 _____ Test #3 _____ Total _____

If you skip the final exam, your course grade will be _____

1. (20 points) A population can be modeled by the following discrete dynamical system

$$u(n) = u(n - 1) + R \cdot u(n - 1)$$

where R is a function of the population u and is shown in the following graph.



- (a) Determine the intrinsic growth rate for this population?
- (b) Find all 3 equilibrium values for this population.
- (c) Sketch a rough graph of the population as a function of time, being sure to show each equilibrium value clearly and being sure to show what happens to any initial populations which are above or below each positive equilibrium value.

(d) Determine the minimum viable population.

(e) Find a formula for R as a function of u given that its graph is a parabola.

(f) If $u(0) = 20$, then what is the value of $u(8)$?

2. (10 points) Find an explicit solution to the following initial value problem.

$$\frac{dw}{dt} = 8e^{-2t}, \quad w(0) = 10$$

3. (10 points) Find an explicit solution to the following initial value problem.

$$\frac{dq}{dr} = 0.2q, \quad q(0) = 150$$

4. (10 points) Find an explicit solution to the following initial value problem.

$$\frac{dp}{dt} = e^{-p}, \quad p(0) = \ln 2$$

5. (10 points) The discrete dynamical system below has an unstable equilibrium value of 0 and a stable equilibrium value of 500. Determine the maximum interval of stability for the stable equilibrium value.

$$u(n) = 1.2u(n-1) - 0.0004u^2(n-1)$$

6. (10 points) In the discrete dynamical system below, find each equilibrium value and determine if it is stable or unstable.

$$u(n) = 0.01u^2(n-1) + 0.3u(n-1) + 10$$

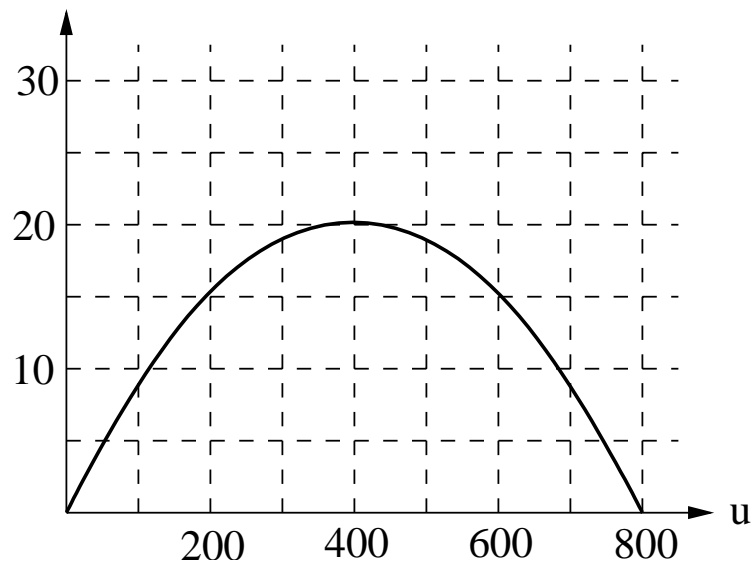
7. (15 points) Suppose that a man-made lake is stocked with 200 fish and that the fish population then grows logistically with an intrinsic growth rate of 8.5% and a carrying capacity of 800.

(a) Determine a discrete dynamical system with initial value to model this fish population.

(b) Carefully sketch a graph of this fish population as a function of time. Be sure to include specific coordinates along the vertical axis which show the interesting features of this graph.

(c) Carefully sketch a graph of the growth rate of this fish population as a function of population.

8. (15 points) The following graph gives the growth of a fish population in a year as a function of its population size.



- (a) Estimate the stable equilibrium population if there is a constant yearly harvest of 15 fish.
- (b) Estimate the minimum viable population if there is a constant yearly harvest of 15 fish.
- (c) Use this graph to estimate the maximum constant sustainable harvest.
- (d) What is the population size that goes along with the harvest in part (c)?
- (e) What percent of the fish population should be harvested each year to maximize the sustainable harvest?