Math 172 **Quiz 3 Solutions** **Fall 2006**

1. Consider the dynamical system

$$u(n) = 0.3u(n-1) - 0.5v(n-1) + 30$$
$$v(n) = 0.2u(n-1) + v(n-1) - 4$$

(a) Find the equilibrium point.

Solving the equations $E = 0.3E - 0.5F + 30$ and $F = 0.2E + F - 4$, we get $E = 20$ and $F = 32$. So the equilibrium point for (u, v) is $(20, 32)$.

(b) Does the equilibrium point appear to be stable or unstable? Show enough work to justify your answer.

n	u(n)	v(n)
0	8	14
5	30.889	20.173
10	24.031	27.939
15	21.335	30.664
20	20.438	31.562
25	20.144	31.856
30	20.047	31.953
35	20.015	31.985
40	20.005	31.995
45	20.002	31.998
50	20.001	31.999

n	u(n)	v(n)
0	12	96
5	-10.288	64.038
10	9.211	42.844
15	16.437	35.564
20	18.832	33.168
25	19.617	32.383
30	19.875	32.125
35	19.959	32.041
40	19.987	32.013
45	19.996	32.004
50	19.999	32.001

n	u(n)	v(n)
0	82	0
5	25.495	27.443
10	21.337	30.692
15	20.424	31.577
20	20.138	31.862
25	20.045	31.955
30	20.015	31.985
35	20.005	31.995
40	20.002	31.998
45	20.001	31.999
50	20.000	32.000

n	u(n)	v(n)
0	213	147
5	-68.925	130.550
10	-13.894	66.195
15	8.745	43.264
20	16.307	35.693
25	18.790	33.210
30	19.603	32.397
35	19.870	32.130
40	19.957	32.043
45	19.986	32.014
50	19.995	32.005

The equilibrium point $(20, 32)$ appears to be stable.

(c) For $u(0) = 10$ and $v(0) = 20$, determine the rate at which $v(n)$ goes toward infinity (if unstable) or goes toward equilibrium (if stable). Show the calculations you made to find the rate.

First we obtain a table of values for the functions u and v given our initial values.

n	u(n)	v(n)
0	10	20
1	23	18
2	27.9	18.6
3	29.07	20.18
4	28.631	21.994
5	27.5923	23.7202
6	26.41759	25.23866
7	25.305947	26.522178
8	24.3306951	27.5833674
9	23.50752483	28.44950642
10	22.827504239	29.151011386

n	u(n)	v(n)
11	22.2727455787	29.7165122338
12	21.8235675567	30.1710613495
13	21.4615395922	30.5357748609
14	21.1705744472	30.8280827793
15	20.9371309445	31.0621976688
16	20.750040449	31.2496238577
17	20.6002002059	31.3996319475
18	20.480244088	31.5196719886
19	20.3842372321	31.6157208062
20	20.3074107665	31.6925682527

Using the values for the function v shown in the table, we obtain the following:

$\frac{v(1)-32}{v(0)-32} \approx 1.16667$

$\frac{v(2)-32}{v(1)-32} \approx 0.95714$

$\frac{v(3)-32}{v(2)-32} \approx 0.88209$

$\frac{v(4)-32}{v(3)-32} \approx 0.84653$

$\frac{v(5)-32}{v(4)-32} \approx 0.82748$

$\frac{v(6)-32}{v(5)-32} \approx 0.81661$

$\frac{v(7)-32}{v(6)-32} \approx 0.81017$

$\frac{v(8)-32}{v(7)-32} \approx 0.80628$

$\frac{v(9)-32}{v(8)-32} \approx 0.80389$

$\frac{v(10)-32}{v(9)-32} \approx 0.80242$

$\frac{v(11)-32}{v(10)-32} \approx 0.80151$

$\frac{v(12)-32}{v(11)-32} \approx 0.80094$

$\frac{v(13)-32}{v(12)-32} \approx 0.80059$

$\frac{v(14)-32}{v(13)-32} \approx 0.80037$

$\frac{v(15)-32}{v(14)-32} \approx 0.80023$

$\frac{v(16)-32}{v(15)-32} \approx 0.80014$

$\frac{v(17)-32}{v(16)-32} \approx 0.80009$

$\frac{v(18)-32}{v(17)-32} \approx 0.80006$

$\frac{v(19)-32}{v(18)-32} \approx 0.80003$

$\frac{v(20)-32}{v(19)-32} \approx 0.80002$

It appears that $\lim_{n\to\infty} \dfrac{u(n)-32}{u(n-1)-32} = 0.8$

So for large enough n, $u(n)$ is getting approximately 20% closer to its equilibrium value each day (assuming n is measured in days).

2. For the following dynamical system, there is no equilibrium point, but the values for $u(n)$ (eventually) change by approximately the same amount.

$$u(n) = 0.9u(n-1) + 0.2v(n-1) + 600$$
$$v(n) = 0.1u(n-1) + 0.8v(n-1) + 400$$

(a) What is that approximate amount by which $u(n)$ eventually changes?

Since no initial values are given, you must first select values for $u(0)$ and $v(0)$. Regardless of the initial values chosen you should find that $\lim_{n\to\infty} (u(n) - u(n-1)) \approx 666.7$

So for large enough n, $u(n)$ is increasing by approximately 666.7 each day (assuming n is measured in days).

(b) Does $u(n)$ appear to be more linear or exponential for large n?

For large enough n, $u(n)$ appears to be linear with an approximate slope of 666.7.