

Name _____

1. A biologist studied the growth of a rabbit population in a field. She let $f(t)$ represent the number of rabbits t weeks from the start of her research. Suppose that $f'(9) = 8$. Which of the following sentences must be true?

The correct answer is: (c) Nine weeks after the start of her research, the rabbit population was increasing by eight rabbits per week.

2. On the graph of $y = 4x^2 - 300$, what is the slope of the curve at $x = 10$?

We note that $y' = 8x$ and let $x = 10$ to obtain that the correct answer is: 80

3. If $y = e^{5x}$, then

$$\frac{dy}{dx} = 5e^{5x}$$

4. Suppose that 100 rabbits were released on an island that had no previous rabbits. Let R denote the rabbit population t months after they were released. The rabbit population grows at a rate which is proportional to the population size itself, where the constant of proportionality is 0.05 (i.e. a continuous growth rate of 5% per month). Write down a differential equation with initial condition to model the growth of this rabbit population.

$$\frac{dR}{dt} = 0.05R \quad \text{and} \quad R(0) = 100$$

5. Given the following initial value problem, use Euler's Method with $\Delta t = 2$ to estimate $w(6)$.

$$\frac{dw}{dt} = \ln(w + 1), \quad w(0) = 10$$

t_{old}	w_{old}	w'_{old}	$w_{new} \approx w_{old} + w'_{old} \cdot \Delta t$
0	10	2.4	14.8
2	14.8	2.8	20.3
4	20.3	3.1	26.4
6	26.4		

Using Euler's Method with $\Delta t = 2$, we obtain the estimate $w(6) \approx 26.4$

6. Suppose y is a function of t which satisfies the differential equation

$$\frac{dy}{dt} = \frac{4(y - 5)(y - 20)}{21}$$

On one set of axes, sketch 5 plausible graphs for y given these 5 initial values: $y(0) = 0$, $y(0) = 5$, $y(0) = 10$, $y(0) = 20$, $y(0) = 25$.

y is increasing if $y < 5$ or $y > 20$

y is decreasing if $5 < y < 20$

$y = 5$ is a stable equilibrium point and $y = 20$ is an unstable equilibrium point

See the graph in class on Monday