- LINEAR GROWTH ($m$ = slope, $P_0$ = initial value)

| Type | Linear Model | Explicit Solution |
|---|---|---|
| continuous | $\frac{dP}{dt} = m$ | $P = mt + P_0$ |
| discrete | $P(n) = P(n-1) + m$ | $P = mt + P_0$ |

- EXPONENTIAL GROWTH ($r$ = growth rate, $P_0$ = initial value)

| Type | Exponential Model | Explicit Solution |
|---|---|---|
| continuous | $\frac{dP}{dt} = rP$ | $P = P_0 e^{rt}$ |
| discrete | $P(n) = P(n-1) + rP(n-1)$ | $P = P_0 (1+r)^n$ |

- LOGISTIC GROWTH ($r$ = growth rate, $k$ = carrying capacity, $P_0$ = initial value)

| Type | Logistic Model |
|---|---|
| continuous | $\frac{dP}{dt} = rP\left(1 - \frac{P}{k}\right)$ |
| discrete | $P(n) = P(n-1) + rP(n-1)\left(1 - \frac{P(n-1)}{k}\right)$ |

1. If $P$ represents some population $t$ years from now, and $\frac{dP}{dt} = 20$, then which of the following statements is correct?

   (a) $\boxed{P \text{ grows linearly by 20 people per year.}}$

   (b) $P$ grows linearly by 120 people per year.

   (c) $P$ grows exponentially by 20% per year.

   (d) $P$ grows logistically with a carrying capacity of 20.

2. If $P$ represents some population $n$ years from now, and $P(n) = 1.2P(n-1)$, then which of the following statements is correct?

   (a) $P$ grows linearly by 20 people per year.

   (b) $P$ grows linearly by 120 people per year.

   (c) $\boxed{P \text{ grows exponentially by 20\% per year.}}$

   (d) $P$ grows logistically with a carrying capacity of 120.

3. The population of a city was 5000 in 1980. Since then the population has been increasing by 100 people per year.

   (a) Determine a discrete dynamical system with initial value to model the city's population.

   $\boxed{P(n) = P(n-1) + 100 \qquad \text{and} \qquad P(0) = 5000}$

   (b) Determine a differential equation with initial value to model the city's population.

   $\boxed{\frac{dP}{dt} = 100 \qquad \text{and} \qquad P(0) = 5000}$

   (c) Determine an explicit formula for the city's population.

   $\boxed{P = 100t + 5000}$

   (d) What does your model predict for the city's population in the year 2000?

   $\boxed{7000}$

   (e) When does your model predict the population will have reached 10000?

   $\boxed{\text{In the year 2030}}$

4. An initial deposit of \$200 is made into an account with an annual percentage rate (APR) of 3% compounded annually.

   (a) Determine a discrete dynamical system with initial value to model the amount of money in this account.

   $\boxed{A(n) = 1.03A(n-1) \qquad \text{and} \qquad A(0) = 200}$

   (b) Determine an explicit formula for the amount of money in this account.

   $\boxed{A = 200(1.03)^n}$

   (c) How much money will the account hold 8 years after the initial deposit?

   $\boxed{\$253.35}$

(d) How long will it take until the balance in this account is \$500?

31.0 years

5. An initial deposit of 200 is made into an account with an annual percentage rate (APR) of 3% compounded continuously.

(a) Determine a differential equation with initial value to model the amount of money in this account.

$\frac{dA}{dt} = 0.03A$ and $A(0) = 200$

(b) Determine an explicit formula for the amount of money in this account.

$A = 200e^{0.03t}$

(c) How much money will the account hold 8 years after the initial deposit?

\$254.25

(d) How long will it take until the balance in this account is \$500?

30.5 years

6. Why did I suggest using a discrete dynamical system for problem (4), but a differential equation for problem (5)?

The interest was compounded annually for problem (4) but continuously for problem (5)

7. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.

(a) Determine a discrete dynamical system with initial value to model the number of fish in this pond.

If we let $F(n)$ represent the number of fish $n$ years after 1970, then

$F(n) = 1.05F(n-1)$ and $F(0) = 300$

(b) Enter this system into your calculator to make a table of values for the number of fish in the pond each year from 1970 to 1980.

| $n$ | $F(n)$ |
|---|---|
| 0 | 300.00 |
| 1 | 315.00 |
| 2 | 330.75 |
| 3 | 347.29 |
| 4 | 364.65 |
| 5 | 382.88 |
| 6 | 402.03 |
| 7 | 422.13 |
| 8 | 443.24 |
| 9 | 465.40 |
| 10 | 488.67 |

(c) Determine an explicit formula for the number of fish in the pond.

$F(n) = 300(1.05)^n$

(d) When will the fish population reach 750?

18.8 years later

8. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.

   (a) Determine a differential equation with initial value to model the number of fish in this pond.

   $\frac{dF}{dt} = 0.05F$ and $F(0) = 300$

   (b) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of fish in the pond each year from 1970 to 1980.

   Since $\Delta t = 1$, our table will have the same values as the discrete dynamical system shown in #7b.

   (c) Determine an explicit formula for the number of fish in the pond.

   $F(t) = 300e^{0.05t}$

   (d) When will the fish population reach 750?

   18.3 years later

9. Which model was best to use for the fish population – the discrete dynamical system or the differential equation? Why?

   discussed in class

10. There are currently 5000 deer in a forest. Suppose the population of deer grows logistically with an intrinsic growth rate of 6% and a carrying capacity of 20,000.

   (a) Sketch a rough graph of the deer population.

   graph shown in class

   (b) Determine a discrete dynamical system with initial value to model the deer population.

   If we let $D(n)$ represent the number of deer $n$ years from now, then

   $D(n) = D(n-1) + 0.06D(n-1)\left(1 - \frac{D(n-1)}{20000}\right)$ and $D(0) = 5000$

   (c) Make a table of values for the number of deer your discrete model predicts for the next 4 years.

| $n$ | $D(n)$ |
|---|---|
| 0 | 5000.0 |
| 1 | 5225.0 |
| 2 | 5456.6 |
| 3 | 5694.7 |
| 4 | 5939.1 |

   (d) Determine a differential equation with initial value to model the deer population.

   $\frac{dD}{dt} = 0.06D\left(1 - \frac{D}{20000}\right)$ and $D(0) = 5000$

(e) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.

| $t_{old}$ | $D_{old}$ | $D'_{old}$ | $D_{new} \approx D_{old} + D'_{old} \cdot \Delta t$ |
|---|---|---|---|
| 0 | 5000.0 | 225.0 | 5225.0 |
| 1 | 5225.0 | 231.6 | 5456.6 |
| 2 | 5456.6 | 238.1 | 5694.7 |
| 3 | 5694.7 | 244.4 | 5939.1 |
| 4 | 5939.1 | | |

Note that since $\Delta t = 1$, our table has the same values as the discrete dynamical system shown in part c.

(f) Use Euler's Method with $\Delta t = 0.5$ to make a table of values for the number of deer your continous model predicts for the next 4 years.

| $t_{old}$ | $D_{old}$ | $D'_{old}$ | $D_{new} \approx D_{old} + D'_{old} \cdot \Delta t$ |
|---|---|---|---|
| 0.0 | 5000.00 | 225.00 | 5112.50 |
| 0.5 | 5112.50 | 228.34 | 5226.67 |
| 1.0 | 5226.67 | 231.65 | 5342.50 |
| 1.5 | 5342.50 | 234.92 | 5459.96 |
| 2.0 | 5459.96 | 238.16 | 5579.04 |
| 2.5 | 5579.04 | 241.37 | 5699.73 |
| 3.0 | 5699.73 | 244.52 | 5821.99 |
| 3.5 | 5821.99 | 247.63 | 5945.81 |
| 4.0 | 5945.81 | | |

(g) How close are the approximations for discrete and continuous models?

discussed in class