Math 172 **Test 3 (part 1 – no calculators) Solutions** **Spring 2006**

1. (8 points) Evaluate the following indefinite integral.

$$\int \left(2e^{5t} + e^{-t}\right)\ dt$$

$\dfrac{2}{5}e^{5t} - e^{-t} + C$

2. (8 points) Find an explicit solution to the following initial value problem.

$$\frac{dw}{dt} = 0.3w, \qquad w(0) = 40$$

$w = 40e^{0.3t}$

3. (8 points) Find an explicit solution to the following initial value problem.

$$\frac{dq}{dr} = r^2, \qquad q(0) = 2$$

$q = \frac{1}{3}r^3 + 2$

4. (8 points) Find an explicit solution to the following initial value problem.

$$\frac{dy}{dx} = \frac{2x}{9y^2} \qquad y(0) = 1$$

Separating variables and then integrating we get $\int 9y^2\ dy = \int 2x\ dx$. So $3y^3 = x^2 + C$. Using our initial value $y(0) = 1$ we find that $C = 3$. So $3y^3 = x^2 + 3$. Solving for y we obtain our solution $y = \sqrt[3]{(x^2+3)/3}$.

Math 172 **Test 3 (part 2 – calculators allowed) Solutions** **Spring 2006**

1. Suppose that an animal population grows logistically with an intrinsic growth rate of 10% and a carrying capacity of 400.

   (a) (5 points) Carefully sketch a graph of the growth rate for this population as a function of the population itself. Be sure to clearly label the values for all intercepts.

   The graph was discussed in class – it is a line with vertical intercept at 0.1 and horizontal intercept at 400.

   (b) (10 points) Carefully sketch a graph of the population as a function of time beginning with an initial population of 50 animals. Clearly show any long term behavior.

   The graph was discussed in class – the vertical intercept is at 50, it increases towards and levels off at a height of 400 as you move to the right. It has that typical S-shaped curve we have seen for logistic growth.

(c) (10 points) Determine a discrete dynamical system to model this population.

$$u(n) = u(n-1) + 0.1u(n-1)\left(1 - \frac{u(n-1)}{400}\right)$$

2. (10 points) Suppose we have the following discrete dynamical system.

$$u(n) = 0.3u^2(n-1) - 2.6u(n-1) + 9.6$$

Find each equilibrium value and determine whether it is stable or unstable.

There is a stable equilibrium value at 4 and an unstable equilibrium value at 8.

3. (10 points) Suppose we have the following discrete dynamical system.

$$u(n) = 1.4u(n-1) - 0.004u^2(n-1)$$

This system has an unstable equilibrium value at 0 and a stable equilibrium value at 100. Which of the following intervals is the maximum interval of stability for the stable equilibrium value?

The maximum interval of stability is $(0, 350)$.

4. (15 points) A population can be modeled by the following discrete dynamical system

$$u(n) = u(n-1) + R \cdot u(n-1)$$

where R is a function of the population u and is shown in the following graph (see blank copy of test).

(a) What is the intrinsic growth rate for this population?

0.3

(b) Find all 3 equilibrium values for this population.

0, 100, and 900

(c) Sketch a rough graph of the population as a function of time, being sure to show each equilibrium value clearly and being sure to show what happens to any initial populations which are above or below each positive equilibrium value.

The graph was discussed in class – initial values between 0 and 100 move towards 0, initial values between 100 and 900 move toward 900, initial values just above 900 move toward 900.

(d) Does this population have a minimum viable population? If so, determine its value.

Yes, the minimum population is 100.

(e) Find a formula for R and use this to write down the appropriate discrete dynamical system for this population.

$$R = \frac{-0.3}{400^2}(u - 500)^2 + 0.3$$

$$u(n) = u(n-1) + \left(\frac{-0.3}{400^2}\left(u(n-1) - 500\right)^2 + 0.3\right) \cdot u(n-1)$$

5. (8 points) Suppose we have the following discrete dynamical system.

$$u(n) = 1.4u(n-1) - 0.004u^2(n-1) \qquad \text{and} \qquad u(0) = 30$$

For large enough n, how does $u(n)$ approach its equilibrium value of 100 ?

We find that

$$\lim_{n \to \infty} \frac{u(n) - 100}{u(n-1) - 100} = 0.6$$

so it gets approximately 40% closer each time period.