

1. (8 points) Evaluate the following indefinite integral.

$$\int (2e^{5t} + e^{-t}) dt$$

2. (8 points) Find an explicit solution to the following initial value problem.

$$\frac{dw}{dt} = 0.3w, \quad w(0) = 40$$

3. (8 points) Find an explicit solution to the following initial value problem.

$$\frac{dq}{dr} = r^2, \quad q(0) = 2$$

4. (8 points) Find an explicit solution to the following initial value problem.

$$\frac{dy}{dx} = \frac{2x}{9y^2} \quad y(0) = 1$$

2. (10 points) Suppose we have the following discrete dynamical system.

$$u(n) = 0.3u^2(n-1) - 2.6u(n-1) + 9.6$$

Find each equilibrium value and determine whether it is stable or unstable.

3. (10 points) Suppose we have the following discrete dynamical system.

$$u(n) = 1.4u(n-1) - 0.004u^2(n-1)$$

This system has an unstable equilibrium value at 0 and a stable equilibrium value at 100. Which of the following intervals is the maximum interval of stability for the stable equilibrium value?

- (a) (0, 100)
- (b) (0, 250)
- (c) (0, 350)
- (d) (0, 450)
- (e) (100, 250)
- (f) (100, 350)
- (g) (100, 450)

4. (15 points) A population can be modeled by the following discrete dynamical system

$$u(n) = u(n - 1) + R \cdot u(n - 1)$$

where R is a function of the population u and is shown in the following graph.

(a) What is the intrinsic growth rate for this population?

(b) Find all 3 equilibrium values for this population.

(c) Sketch a rough graph of the population as a function of time, being sure to show each equilibrium value clearly and being sure to show what happens to any initial populations which are above or below each positive equilibrium value.

(d) Does this population have a minimum viable population? If so, determine its value.

(e) Find a formula for R and use this to write down the appropriate discrete dynamical system for this population.

5. (8 points) Suppose we have the following discrete dynamical system.

$$u(n) = 1.4u(n-1) - 0.004u^2(n-1) \quad \text{and} \quad u(0) = 30$$

For large enough n , how does $u(n)$ approach its equilibrium value of 100 ?

- (a) It gets approximately 20% closer each time period.
- (b) It gets approximately 30% closer each time period.
- (c) It gets approximately 40% closer each time period.
- (d) It gets approximately 50% closer each time period.
- (e) It gets approximately 60% closer each time period.
- (f) It gets approximately 70% closer each time period.