1. Given the dynamical system $p(n) = 2p(n-1) + 5$ with $p(1) = 5$, find values for $p(2)$, $p(3)$, and $p(4)$.

   Solution:

   $p(2) = 2 \cdot p(1) + 5 = 2 \cdot 5 + 5 = 15$
   $p(3) = 2 \cdot p(2) + 5 = 2 \cdot 15 + 5 = 35$
   $p(4) = 2 \cdot p(3) + 5 = 2 \cdot 35 + 5 = 75$

   So p(2)=15, p(3)=35, and p(4)=75

2. Consider the following dynamical system of two equations.

$$u(n) = u(n-1) + v(n-1) - 2$$
$$v(n) = 2u(n-1) - v(n-1)$$

   If $u(0) = 3$ and $v(0) = 2$, then determine $u(3)$ and $v(3)$.

   Solution:

   $u(1) = u(0) + v(0) - 2 = 3 + 2 - 2 = 3$ and $v(1) = 2 \cdot u(0) - v(0) = 2 \cdot 3 - 2 = 4$
   $u(2) = u(1) + v(1) - 2 = 3 + 4 - 2 = 5$ and $v(2) = 2 \cdot u(1) - v(1) = 2 \cdot 3 - 4 = 2$
   $u(3) = u(2) + v(2) - 2 = 5 + 2 - 2 = 5$ and $v(3) = 2 \cdot u(2) - v(2) = 2 \cdot 5 - 2 = 8$

   So u(3)=5 and v(3)=8

3. Let $h(n)$ represent the height of a stack of n chairs. Each chair by itself is 3 feet high, but when stacked, the height of an existing stack only increases by 8 inches for each additional chair. A pattern doesn't really begin until you actually have one chair, so we won't define $h(0)$ but will start with $h(1) = 3$. Be consistent with your units and do the following:

   (a) Develop a discrete dynamical system for $h(n)$.

   Solution 1 (inches): Let $h(n)$ represent the height in inches of n stacked chairs with $n \geq 1$. Then

   h(n) = h(n-1) + 8 and h(1) = 36

   Solution 2 (feet): Let $h(n)$ represent the height in feet of n stacked chairs with $n \geq 1$. Then

   h(n) = h(n-1) + 2/3 and h(1) = 3

(b) Find an explicit formula for $h(n)$.

Solution 1 (inches): $h(n) = 8(n-1) + 36$ or $h(n) = 8n + 28$

Solution 2 (feet): $h(n) = \frac{2}{3}(n-1) + 3$ or $h(n) = \frac{2}{3}n + \frac{7}{3}$

4. Develop a discrete model in which 80% of some drug in the bloodstream from one day to the next is used up, but the remainder is reinforced with maintenance dose of 40 mg per day. The initial dose is 10 mg.

Solution:

Let $u(n)$ represent the number of milligrams of drug in the bloodstream n days after the initial dose. Then

u(n)=u(n-1) - 0.8u(n-1) + 40 and u(0) = 10

or in simplified form

u(n)=0.2u(n-1) + 40 and u(0) = 10

5. There are 2 drugs, A and B. Let $a(n)$ and $b(n)$ represent the number of milligrams of each drug in the body at the beginning of day n. The body converts 10% of A into B each day and converts 25% of B into A each day. Assume that 800 mg of A and 600 mg of B are consumed each day, and that the body eliminates 100 mg of A and 200 mg of B each day. Begin with $a(0) = 800$ and $b(0) = 600$ and develop a discrete model to represent $a(n)$ and $b(n)$.

Solution:

a(n)=a(n-1) - 0.1a(n-1) + 0.25b(n-1) + 800 - 100

b(n)=b(n-1) + 0.1a(n-1) - 0.25b(n-1) + 600 - 200

a(0)=800 and b(0)=600

or in simplified form

a(n)=0.9a(n-1) + 0.25b(n-1) + 700

b(n)=0.1a(n-1) + 0.75b(n-1) + 400

a(0)=800 and b(0)=600