

1. Recall that smoking a pack of cigarettes per day results in absorbing about 2.7 mg of cadmium each year from smoking. Assume that about 7% of the cadmium is eliminated from the body each year. Find the amount of cadmium in this person's body as a result of smoking for 30 years.

*Solution:*

Let  $u(n)$  represent the number of milligrams of cadmium in a person's body from smoking a pack of cigarettes per day for  $n$  years. Then

$$u(n) = u(n-1) - 0.07u(n-1) + 2.7 \quad \text{and} \quad u(0) = 0$$

or in simplified form

$$u(n) = 0.93u(n-1) + 2.7 \quad \text{and} \quad u(0) = 0$$

This gives  $u(30) \approx \boxed{34.2 \text{ mg}}$

2. Suppose you borrow \$150,000 at an 8.4% annual interest rate compounded monthly to be paid back in monthly payments of \$2000.
- (a) Write down a discrete dynamical system with initial condition to represent the balance of the loan just after each month's payment.

*Solution:*

Let  $a(n)$  represent the amount owed  $n$  years after obtaining the loan. Then

$$a(n) = a(n-1) + (0.084/12)a(n-1) - 2000 \quad \text{and} \quad a(0) = 150,000$$

or in simplified form

$$\boxed{a(n) = 1.007a(n-1) - 2000 \quad \text{and} \quad a(0) = 150,000}$$

- (b) How many months will it take to pay back the loan?

*Solution:*

We find that  $a(106) = 1432.23$  and  $a(107) = -557.74$

It will take  $\boxed{107 \text{ months}}$ , but if each payment is \$2000, then we have overpaid the loan by the \$557.74.

- (c) The last payment will be a bit different than each of the preceding monthly payments. What will be the amount of this last payment?

*Solution:*

$$\$2000 - \$557.74 = \boxed{\$1442.26}.$$

3. Find the equilibrium value for the following dynamical system.

$$u(n) = 1.25u(n - 1) - 7.3$$

*Solution:*

Solving the equation  $E = 1.25E - 7.3$ , we get the equilibrium value  $E = 29.2$ .

4. Find the equilibrium point for the following dynamical system.

$$u(n) = 2u(n - 1) + v(n - 1) + 3$$

$$v(n) = 4u(n - 1) - v(n - 1) + 6$$

*Solution:*

Solving the equations  $E = 2E + F + 3$  and  $F = 4E - F + 6$ , we get the equilibrium point  $(E, F) = (-2, -1)$ .