

1. Consider the dynamical system

$$u(n) = 0.3u(n - 1) - 0.5v(n - 1) + 30$$

$$v(n) = 0.2u(n - 1) + v(n - 1) - 4$$

(a) Find the equilibrium point.

Solving the equations  $E = 0.3E - 0.5F + 30$  and  $F = 0.2E + F - 4$ , we get the equilibrium point  $(E, F) = (20, 32)$ .

(b) Does the equilibrium point appear to be stable or unstable? Show enough work to justify your answer.

n	u(n)	v(n)	n	u(n)	v(n)	n	u(n)	v(n)	n	u(n)	v(n)
0	8	14	0	12	96	0	82	0	0	213	147
5	30.889	20.173	5	-10.288	64.038	5	25.495	27.443	5	-68.925	130.550
10	24.031	27.939	10	9.211	42.844	10	21.337	30.692	10	-13.894	66.195
15	21.335	30.664	15	16.437	35.564	15	20.424	31.577	15	8.745	43.264
20	20.438	31.562	20	18.832	33.168	20	20.138	31.862	20	16.307	35.693
25	20.144	31.856	25	19.617	32.383	25	20.045	31.955	25	18.790	33.210
30	20.047	31.953	30	19.875	32.125	30	20.015	31.985	30	19.603	32.397
35	20.015	31.985	35	19.959	32.041	35	20.005	31.995	35	19.870	32.130
40	20.005	31.995	40	19.987	32.013	40	20.002	31.998	40	19.957	32.043
45	20.002	31.998	45	19.996	32.004	45	20.001	31.999	45	19.986	32.014
50	20.001	31.999	50	19.999	32.001	50	20.000	32.000	50	19.995	32.005

The equilibrium point  $(E, F) = (20, 32)$  appears to be stable.

(c) For  $u(0) = 10$  and  $v(0) = 20$ , determine the rate at which  $v(n)$  goes toward infinity (if unstable) or goes toward equilibrium (if stable). Show the calculations you made to find the rate.

First we obtain a table of values for the functions  $u$  and  $v$  given our initial values.

n	u(n)	v(n)	n	u(n)	v(n)
0	10	20	11	22.2727455787	29.7165122338
1	23	18	12	21.8235675567	30.1710613495
2	27.9	18.6	13	21.4615395922	30.5357748609
3	29.07	20.18	14	21.1705744472	30.8280827793
4	28.631	21.994	15	20.9371309445	31.0621976688
5	27.5923	23.7202	16	20.750040449	31.2496238577
6	26.41759	25.23866	17	20.6002002059	31.3996319475
7	25.305947	26.522178	18	20.480244088	31.5196719886
8	24.3306951	27.5833674	19	20.3842372321	31.6157208062
9	23.50752483	28.44950642	20	20.3074107665	31.6925682527
10	22.827504239	29.151011386			

Using the values for the function  $v$  shown in the table, we obtain the following:

$$\frac{v(1)-32}{v(0)-32} \approx 1.16667$$

$$\frac{v(2)-32}{v(1)-32} \approx 0.95714$$

$$\frac{v(3)-32}{v(2)-32} \approx 0.88209$$

$$\frac{v(4)-32}{v(3)-32} \approx 0.84653$$

$$\frac{v(5)-32}{v(4)-32} \approx 0.82748$$

$$\frac{v(6)-32}{v(5)-32} \approx 0.81661$$

$$\frac{v(7)-32}{v(6)-32} \approx 0.81017$$

$$\frac{v(8)-32}{v(7)-32} \approx 0.80628$$

$$\frac{v(9)-32}{v(8)-32} \approx 0.80389$$

$$\frac{v(10)-32}{v(9)-32} \approx 0.80242$$

$$\frac{v(11)-32}{v(10)-32} \approx 0.80151$$

$$\frac{v(12)-32}{v(11)-32} \approx 0.80094$$

$$\frac{v(13)-32}{v(12)-32} \approx 0.80059$$

$$\frac{v(14)-32}{v(13)-32} \approx 0.80037$$

$$\frac{v(15)-32}{v(14)-32} \approx 0.80023$$

$$\frac{v(16)-32}{v(15)-32} \approx 0.80014$$

$$\frac{v(17)-32}{v(16)-32} \approx 0.80009$$

$$\frac{v(18)-32}{v(17)-32} \approx 0.80006$$

$$\frac{v(19)-32}{v(18)-32} \approx 0.80003$$

$$\frac{v(20)-32}{v(19)-32} \approx 0.80002$$

It appears that  $\lim_{n \rightarrow \infty} \frac{u(n) - 32}{u(n-1) - 32} = 0.8$

So for large enough  $n$ ,  $u(n)$  is getting approximately 20% closer to its equilibrium value each day (assuming  $n$  is measured in days).

2. For the following dynamical system, there is no equilibrium point, but the values for  $u(n)$  (eventually) change by approximately the same amount.

$$u(n) = 0.9u(n-1) + 0.2v(n-1) + 600$$

$$v(n) = 0.1u(n-1) + 0.8v(n-1) + 400$$

- (a) What is that approximate amount by which  $u(n)$  eventually changes?

Since no initial values are given, you must first select values for  $u(0)$  and  $v(0)$ . Regardless of the initial values chosen you should find that  $\lim_{n \rightarrow \infty} u(n) - u(n-1) \approx 666.7$

So for large enough  $n$ ,  $u(n)$  is increasing by approximately 666.7 each day (assuming  $n$  is measured in days).

- (b) Does  $u(n)$  appear to be more linear or exponential for large  $n$ ?

For large enough  $n$ ,  $u(n)$  appears to be linear with an approximate slope of 666.7.