

Name \_\_\_\_\_

1. Given the following initial value problem, use Euler's Method with  $\Delta t = 2$  to estimate  $w(6)$ .

$$\frac{dw}{dt} = \ln(w + 1), \quad w(0) = 10$$

2. Suppose  $y$  is a function of  $t$  which satisfies the differential equation

$$\frac{dy}{dt} = \frac{4(y - 5)(y - 20)}{21}$$

- (a) Sketch a rough graph of  $y$  given that  $y(0) = 15$
- (b) Find all real values of  $y$  for which the quantity  $y$  is increasing.
- (c) Find all real values of  $y$  for which the quantity  $y$  is decreasing.
- (d) For which values of  $y$  is the quantity  $y$  in equilibrium? Determine whether each of these equilibrium values is stable or unstable.

3. Using  $P$  for your dependent variable,  $t$  for your independent variable, and  $k$ ,  $r$ ,  $m$ , or  $C$  for any necessary constants, write down the general form for a differential equation which models each of the following types of growth.

(a) logistic growth

(b) linear growth

(c) exponential growth

4. Suppose that 500 trout are released into a man-made lake which had no trout before. Further suppose that the trout population,  $P$ , grows logistically according to the following differential equation where  $t$  represents the number of years since the initial release of the trout.

$$\frac{dP}{dt} = 0.1P \left( 1 - \frac{P}{2500} \right) \quad \text{and} \quad P(0) = 500$$

(a) As a percentage, what is the intrinsic growth rate of this trout population?

(b) What is the carrying capacity for this trout population?

(c) Sketch a rough graph of this trout population being sure to show any long-term behavior.