Derivative Rules

If n, m, b, c, and a are constants ($a > 0$), then

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}(mx + b) = m$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}(a^x) = \ln a \cdot a^x$
5. $\frac{d}{dx}(e^x) = e^x$
6. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
7. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
8. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
9. $\frac{d}{dx}[cf(x)] = cf'(x)$
10. **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ $\left(\text{also written as } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \right)$
11. **Product Rule:** $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
12. **Quotient Rule:** $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

1. Find derivatives of the following functions. Your final answer should be in the form of an equation where the right hand side is your derivative formula, and the left hand side is the notation for your derivative.

   (a) $h = 5t^3 + t^2/3 + 7t - 2$

   (b) $P(t) = 100$

   (c) $g(r) = \ln r + \sqrt{r}$

   (d) $h(x) = 8/x^5$

   (e) $W(t) = \dfrac{1}{2t^4}$

   (f) $y = \dfrac{4}{\sqrt{x}}$

   (g) $w = x^{-1/3}$

   (h) $h = t + \dfrac{1}{t}$

   (i) $P = 100e^{2t}$

   (j) $y = \ln\left(x^4\right)$

   (k) $w = x^2 e^x$

   (l) $f(x) = (x^2 + 1)^{99}$

   (m) $f(x) = \dfrac{1}{(x+3)^{10}}$

   (n) $u(n) = 100e^{0.5n}$

   (o) $f(x) = 10e^{5-x^2}$

   (p) $v(n) = \dfrac{10}{e^n}$

   (q) $f(t) = 3000(1.02)^t$

   (r) $x = \sqrt{t^3 + 1}$

   (s) $y = e^{\sqrt{x}}$

   (t) $w = 50e^{-0.6t}$

   (u) $y = \dfrac{x^5 + 10x^3 + 1}{x^2}$

   (v) $h = \dfrac{x^2}{x^5 + 10x^3 + 1}$

   (w) $W = \dfrac{e^t}{t^3}$

   (x) $f(x) = x^2 e^{-1.5x}$

   (y) $y = \sqrt{\ln\left(x^3 + 2\right)}$

   (z) $f(x) = \ln\left(\dfrac{\sqrt{x^5}}{x^2}\right)$

2. The population of a town is given by $P(t) = 900e^{-0.05t}$ where t represents the numbers of years since 1950. In the year 1980, what is the town's population and how quickly is it changing (in people per year) at that time?

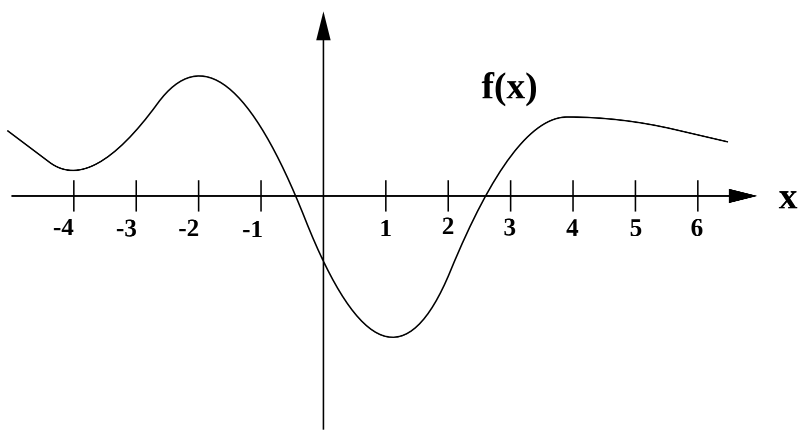
3. Find the equation of the line which is tangent to the graph of $f(x) = x^2 - 4$ at $x = 3$.

4. Find the equation of the line which is tangent to the graph of $P = 10e^{-t}$ at $t = 0$.

5. Dorothy emptied a bucket of water upon the Wicked Witch of the West who immediately began to melt. If the Scarecrow only had a brain, he would calculate that the witch's height could now be given by the function $h(t) = 63(0.91)^t$, where t is measured in seconds since the water was first thrown upon the witch, and $h(t)$ is measured in inches.

   At time $t = 11$ seconds, how tall was the witch and how quickly was her height changing? Each answer should be correctly rounded off to one place after the decimal point and include proper units.

6. Use the graph of $f(x)$ given below to answer the following questions.



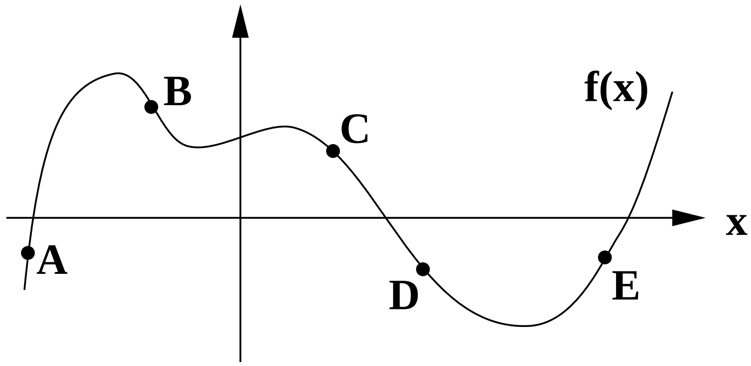
   (a) Which of the following quantities has the largest positive value:

   $f(-4)$, $f(-2)$, $f(0)$, $f(2)$, $f(4)$, or $f(6)$?

   (b) Which of the following quantities has the largest positive value:

   $f'(-4)$, $f'(-2)$, $f'(0)$, $f'(2)$, $f'(4)$, or $f'(6)$?

7. State whether the derivative of the function graphed below is positive, negative, or zero at each of the labeled points.



8. A man lives in a high-rise apartment building. He leans out from one of his apartment windows and throws a ball upward. Between the time that the ball is thrown and the time that the ball hits the ground, the height of the ball is given by the formula $h(t) = -16t^2 + 96t + 160$, where t is the number of seconds since the ball is first thrown and $h(t)$ is measured in feet above ground-level.

   (a) What is the average velocity of the ball during the first 2 seconds?

   (b) Approximate the instantaneous velocity of the ball at $t = 1.5$ seconds.

   (c) When does the ball reach its maximum height?

   (d) What is the ball's maximum height?

   (e) When does the ball hit the ground? Give your answer to at least one decimal place.

   (f) How fast is the ball going when it hits the ground?

9. A can of soda is taken out of the refrigerator and brought outside on a hot summer day. Suppose that $f(t) = 90 - 54e^{-0.2t}$ gives the temperature, in degrees Fahrenheit ($^\circ F$), of the can of soda t minutes after it is taken out of the refrigerator.

   (a) What is the temperature of the can of soda when it is first taken out of the refrigerator?

   (b) How quickly is the soda's temperature increasing the moment it is first taken out of the refrigerator?

   (c) Precisely 6 minutes after the soda is first taken out of the refrigerator, how quickly is its temperature increasing? Give your answer to the nearest two decimal places.

   (d) From the time the soda is taken out of the refrigerator, how long does it take before its temperature reaches $82^\circ F$? Give your answer to the nearest minute.

10. After a weekend away from school, a student carrying a flu virus returned to an isolated college campus. The virus spread and the total number of infected students t days after the student returned to campus can be approximated by $f(t)$.

(a) Suppose that $f(6) \approx 180$. Which of the following choices best describes what this means in practical terms?

(a) Six days after the student returned to campus, there were a total of 180 students infected with the flu virus.

(b) Six days after the student returned to campus, the number of infected students was increasing by 180 students per day.

(c) During the first 6 days after the student returned to campus, the number of infected students was increasing at an average rate of 180 students per day.

(d) The flu lasted for 6 days. A total of 180 students were infected each day.

(e) Every 6 days, 180 more students came down with the flu.

(b) Suppose that $f'(6) \approx 10$. Which of the following choices best describes what this means in practical terms?

(a) Six days after the student returned to campus, there were a total of 10 students infected with the flu virus.

(b) Six days after the student returned to campus, the number of infected students was increasing by 10 students per day.

(c) During the first 6 days after the student returned to campus, the number of infected students was increasing at an average rate of 10 students per day.

(d) The flu lasted for 6 days. A total of 10 students were infected each day.

(e) Every 6 days, 10 more students came down with the flu.

(c) Given that $f(6) \approx 180$ and $f'(6) \approx 10$, approximate the total number of students that were infected with the flu virus 7 days after this student returned to campus?

(a) 250

(b) 240

(c) 210

(d) 190

(e) 70

11. My cat's weight (in pounds) is given by $W(t)$, where t represents the number of years since her birth.

(a) Given that $W(13) = 14$ and $W'(13) = 0.25$, explain what all of this means in practical terms. Your final answer should be in the form of one or more English sentences which can be easily understood by a person who knows very little math. You should especially avoid calculus terms such as derivative, rate of change, function, slope, tangent line, etc.

(b) Using the information in part (a), what do you estimate the cat's weight to be at the age of 15?