Math 172 Test 1 February 21, 2007

Name SOLUTIONS

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your ID when you turn in your test.

#1 (5 points) _____

#2 (10 points) _____

#3 (15 points) _____

#4 (5 points) _____

#5 (5 points) _____

#6 (15 points) _____

#7 (10 points) _____

#8 (10 points) _____

#9 (15 points) _____

#10 (10 points) _____

Total (100 points) _____

1. (5 points) Consider the following discrete dynamical system.

$$u(n) = 0.6u(n-1) + 30$$

If $u(0) = 20$, then what is the value of $u(2)$?

55.2

2. (10 points) A doctor prescribes an initial dose of 100 milligrams of some drug to be followed by a maintenance dose of 50 milligrams each day. Suppose that 20% of the amount of this drug in the bloodstream is eliminated via the kidneys each day. Let $u(n)$ represent the number of milligrams of this drug in the patient's body n days after the initial dose. Determine a discrete dynamical system along with an initial value for $u(n)$.

$u(n) = 0.8u(n-1) + 50$

$u(0) = 100$

3. (15 points) Suppose that $u(n)$ and $v(n)$ represent the number of milligrams of drugs U and V, respectively, in the bloodstream n days after an initial dose of each. The discrete dynamical system for this is shown below.

$$u(n) = 0.3u(n-1) - 0.5v(n-1) + 30$$
$$v(n) = 0.2u(n-1) + v(n-1) - 4$$
$$u(0) = 10$$
$$v(0) = 20$$

(a) How many milligrams of each drug are in the bloodstream 5 days after the initial dose?

$u(5) = 27.6$ mg

$v(5) = 23.7$ mg

(b) After the initial dose, it takes between 11 and 12 days for the amount of drug V in the bloodstream to reach a level of 30 milligrams.

(c) Assuming that the patient is to continue taking this drug for a long time, what was the doctor's **target goal** for the desired amount of each drug in the bloodstream?

20 mg FOR U

32 mg FOR V

4. (5 points) Suppose $p(n)$ represents some population n years from now, and that this population is modeled by the following discrete dynamical system.

$$p(n) = p(n-1) + 40$$
$$p(0) = 60$$

Which one of the following statements follows from this model?

(a) The population will increase by 60% per year.
(b) The population will decrease by 60% per year.
(c) The population will increase by 60 people per year.
(d) The population will decrease by 60 people per year.
(e) The population will increase by 40% per year.
(f) The population will decrease by 40% per year.
(g) The population will increase by 40 people per year.
(h) The population will decrease by 40 people per year.

5. (5 points) Suppose $a(n)$ represents the number of milligrams of some drug in the bloodstream n hours from now, and that the amount of this drug in the bloodstream is modeled by the following discrete dynamical system.

$$a(n) = 0.75a(n-1)$$
$$a(0) = 50$$

Which one of the following statements follows from this model?

(a) The amount of drug in the bloodstream will increase by 25% per hour.
(b) The amount of drug in the bloodstream will decrease by 25% per hour.
(c) The amount of drug in the bloodstream will increase by 25 milligrams per hour.
(d) The amount of drug in the bloodstream will decrease by 25 milligrams per hour.
(e) The amount of drug in the bloodstream will increase by 50% per hour.
(f) The amount of drug in the bloodstream will decrease by 50% per hour.
(g) The amount of drug in the bloodstream will increase by 50 milligrams per hour.
(h) The amount of drug in the bloodstream will decrease by 50 milligrams per hour.
(i) The amount of drug in the bloodstream will increase by 75% per hour.
(j) The amount of drug in the bloodstream will decrease by 75% per hour.
(k) The amount of drug in the bloodstream will increase by 75 milligrams per hour.
(l) The amount of drug in the bloodstream will decrease by 75 milligrams per hour.

6. (15 points) Consider the following discrete dynamical system where n is measured in days.

$$u(n) = 0.95u(n-1) + 30 \text{ and } u(0) = 100$$

(a) Determine the equilibrium value for this dynamical system.

$E = 0.95E + 30$

$0.05E = 30$

$E = 600$

(b) Is the equilibrium value stable or unstable?

i. If the equilibrium value is stable, then by what percentage is $u(n)$ is moving toward its equilibrium value each day?

$$\lim_{n \to \infty} \frac{u(n) - 600}{u(n-1) - 600} = 0.95$$

5%

ii. If the equilibrium value is unstable, then by what percentage is $u(n)$ is moving away from its equilibrium value each day?

7. (10 points) We are given the following discrete dynamical system.

$$u(n) = 0.7u(n-1) + 0.1v(n-1) + 50$$
$$v(n) = 0.3u(n-1) + 0.9v(n-1) + 80$$

Although the function $u(n)$ is ~~not~~ linear, as n increases the function starts to look like a line with slope 32.5.

$$\lim_{n \to \infty} (u(n) - u(n-1)) = 32.5$$

8. (10 points) Iodine-131 is a radioactive isotope used in the treatment of hyperthyroid. It is known that 8.3% of this substance decays each day. Due to a strike at UPS, it took 20 days for a shipment of Iodine-131 to be sent from the producer to a hospital. What percentage of the original amount shipped actually arrived at the hospital?

$u(n) = c(0.917)^n$

$u(20) = c(0.917)^{20}$

$u(20) \approx c(0.17676)$

17.676 %

9. (15 points) Find an explicit formula for an expression which satisfies each of the following discrete dynamical systems.

(a) $u(n) = u(n-1) - 5$ and $u(0) = 40$

$u(n) = -5n + 40$

(b) $u(n) = 1.2u(n-1)$ and $u(0) = 40$

$$u(n) = 40(1.2)^n$$

(c) $u(n) = 1.2u(n-1) - 5$ and $u(0) = 40$

$$E = 1.2E - 5 \Rightarrow E = 25$$

SO $u(n) = c(1.2)^n + 25$

$u(0) = 40 \Rightarrow 40 = c(1.2)^0 + 25$

$\Rightarrow c = 15$

THUS $u(n) = 15(1.2)^n + 25$

10. (10 points) Suppose you borrow \$25,000 at a 7.2% annual interest rate compounded monthly to be paid back in monthly payments of \$500.

(a) Write down a discrete dynamical system with initial condition to represent the balance owed on the loan after making n monthly payments.

$\frac{0.072}{12} = 0.006$

$u(n) = 1.006u(n-1) - 500$

$u(0) = 25000$

(b) How many months does it take to pay back this loan?

EITHER

59 MONTHS WITH 59TH PAYMENT EQUAL TO \$810.48

OR

60 MONTHS WITH 60TH PAYMENT EQUAL TO \$312.34