YELLOW/BLUE VERSION

Math 172 Test 2 (part 1 – no calculators) March 28, 2007

Name SOLUTIONS

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your ID when you turn in your test.

| | |
|---|---|
| #1 (5 points) | |
| #2 (9 points) | |
| #3 (6 points) | |
| #4 (10 points) | |
| #5 (10 points) | |
| #6 (10 points) | |
| #7 (10 points) | |
| #8 (10 points) | |
| #9 (10 points) | |
| #10 (10 points) | |
| #11 (10 points) | |
| Total (100 points) | |

1. (5 points) Ralph Howard purchased some guppies for his new fish tank. They reproduced many times and Ralph noted that the total number of guppies could be approximated by the function $g(t) = t^2 + 30$, where $t$ represents the number of months since his original purchase. Precisely five months after his original purchase, the total number of guppies in his fish tank are increasing by

(a) 55 guppies per month

(b) 40 guppies per month

(c) 30 guppies per month

(d) 25 guppies per month

(e) 10 guppies per month

(f) 5 guppies per month

2. (3 points each) Find derivatives of each of the following functions. Using Leibniz notation (i.e. $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), put your final answer in the form of an equation where the right hand side is your derivative formula, and the left hand side is the notation for your derivative.

(a) $h = 5r^3 + \ln(r)$

$$\frac{dh}{dr} = 15r^2 + \frac{1}{r}$$

(b) $y = \sqrt{x^3 - 5x + 7}$

$$\frac{dy}{dx} = \frac{1}{2}(x^3 - 5x + 7)^{-1/2} \cdot (3x^2 - 5)$$

(c) $w = t^5 e^{-t}$

$$\frac{dw}{dt} = 5t^4 e^{-t} - t^5 e^{-t}$$

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3. (6 points) The number of rabbits in a field after $t$ months of observation is approximated by the formula $R(t) = 100e^{0.03t}$. At what time $t$ is the rabbit population increasing by 30 rabbits per month?

$R'(t) = 3e^{0.03t}$

$30 = 3e^{0.03t}$

$10 = e^{0.03t}$

$\ln(10) = 0.03t$

$t = \frac{\ln(10)}{0.03}$

$t \approx 76.75$ months

4. (10 points) Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 10(P-6)(2P-5)\left(P^2-9\right)\left(P^2+4\right)$$

$6, \frac{5}{2}, -3, 3$

5. (10 points) Given the following initial value problem, use Euler's Method with $\Delta t = 3$ to make an estimate for $P(9)$.

$$\frac{dP}{dt} = 0.05P + 3 \qquad \text{and} \qquad P(0) = 100$$

| $t$ | $P$ | $\frac{dP}{dt}$ | $P_{next} \approx P_{current} + \frac{dP}{dt} \cdot \Delta t$ |
|---|---|---|---|
| 0 | 100 | 8 | $100 + 8(3)$ |
| 3 | 124 | 9.2 | $124 + 9.2(3)$ |
| 6 | 151.6 | 10.58 | $151.6 + 10.58(3)$ |
| 9 | 183.34 | | |

$P(9) \approx 183.34$

6. (10 points) Suppose $y$ is a function of $t$ which satisfies the differential equation below.

$$\frac{dy}{dt} = 0.25y(y-20)^2(y-40)(60-y)$$

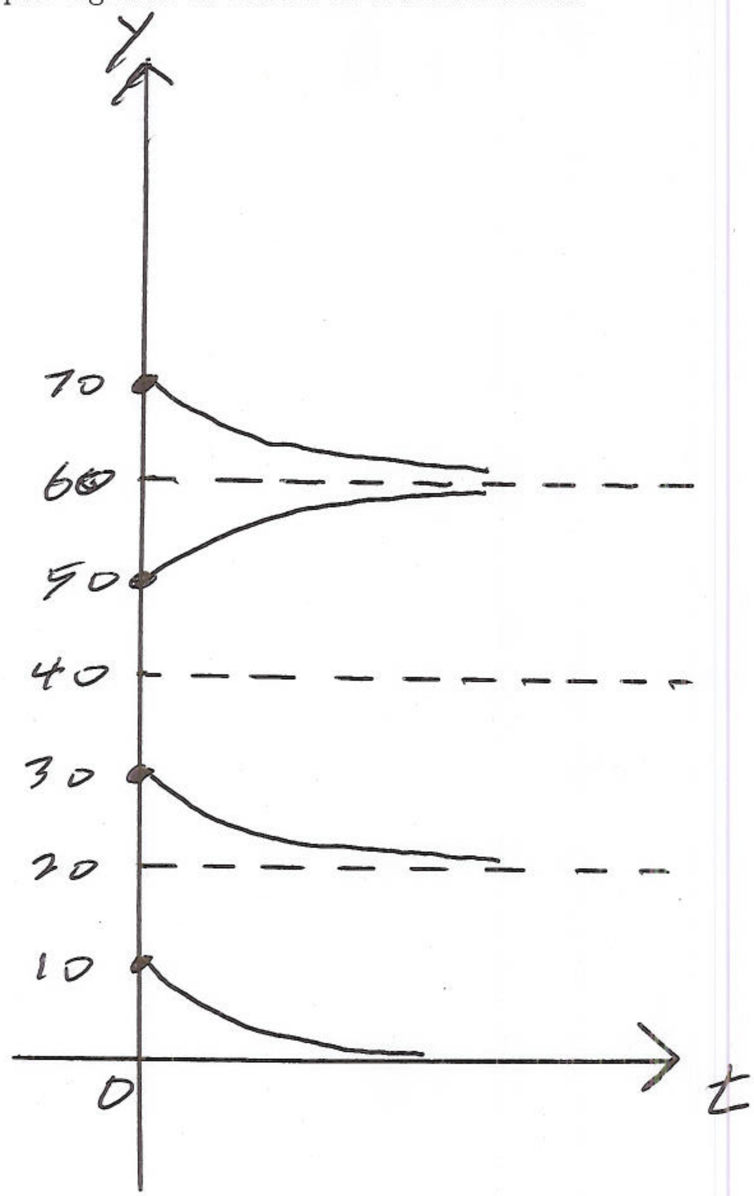
Sketch plausible graphs for $y$ as a function of $t$ given each initial value below. Your graphs should clearly show if the $y$-values approach any particular values (i.e. horizontal asymptotes). You should draw all four graphs together on one set of coordinate axes.

(a) $y(0) = 10$

(b) $y(0) = 30$

(c) $y(0) = 50$

(d) $y(0) = 70$



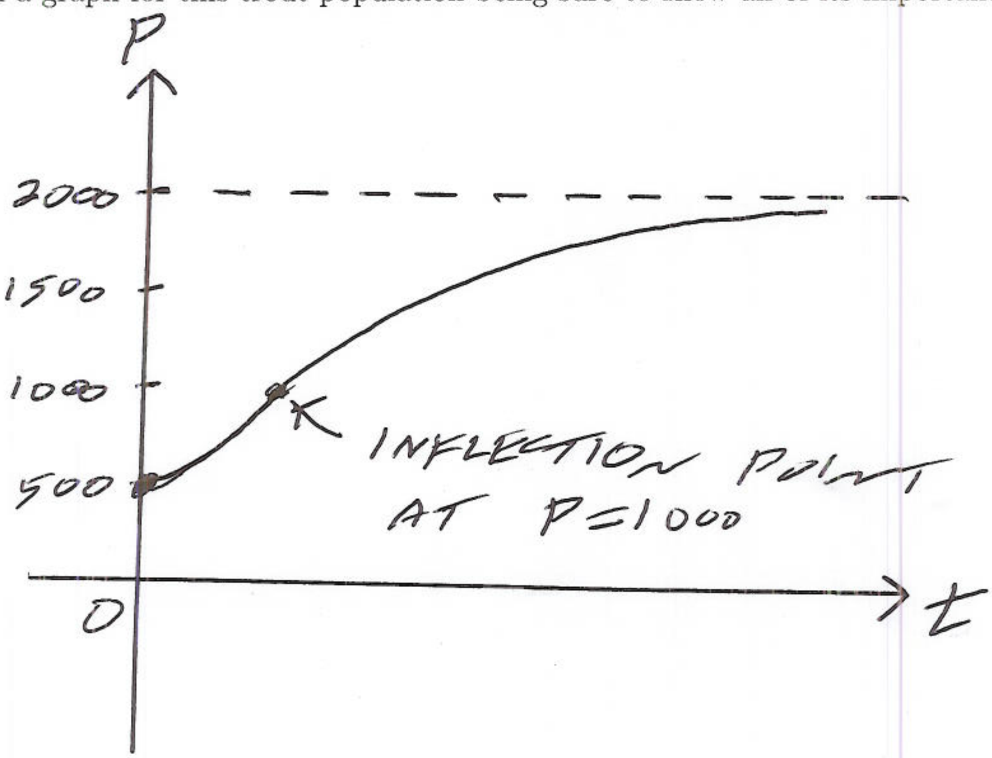
7. (10 points) Suppose that 500 trout are released into a man-made lake which had no trout before. Further suppose that the trout population grows logistically with an instrinsic growth rate of 8.5% and a carrying capacity of 2000 trout.

(a) Determine ~~a differential equation with initial condition~~ to model this trout population.

$$\frac{dP}{dt} = 0.085P\left(1 - \frac{P}{2000}\right)$$

$$P(0) = 500$$

(b) Sketch a graph for this trout population being sure to show all of its important features.



8. (10 points) A tree is currently 8 feet tall and expected to grow 1 foot per year from now on.

(a) Determine a differential equation with initial condition to model this tree's height.

$$\frac{dh}{dt} = 1$$

$$h(0) = 8$$

(b) Determine a discrete ~~dynamical system~~ with initial condition to model this tree's height.

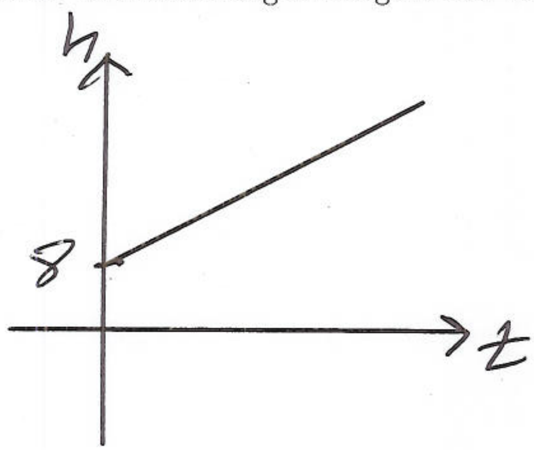
$$h(n) = h(n-1) + 1$$

$$h(0) = 8$$

(c) Find an explicit formula for this tree's height.

$$h = t + 8$$

(d) Sketch a graph for this tree's height being sure to show all of its important features.




A LINE WITH SLOPE = 1 AND VERTICAL INTERCEPT = 8

9. (10 points) Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after $n$ days. Assume that each day, the kidneys remove 15% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{4}{6+u(n-1)}$$

Finally, assume that each day, the person takes a dose of 100 mg of this chemical. Develop a dynamical system for $u(n)$. You do not need an initial value.

$$u(n) = u(n-1) - 0.15u(n-1) - \frac{4u(n-1)}{6+u(n-1)} + 100$$

10. (10 points) For Beth's metabolism, the dynamical system modeling her elimination of alcohol is

$$a(n) = a(n-1) - \frac{9.5a(n-1)}{4.5 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in her bloodstream after $n$ hours of drinking $d$ grams of alcohol per hour. For Beth's weight, 37 grams of alcohol in the bloodstream represents a blood alcohol level of 0.08. Correct to one place after the decimal point, what is the largest number of grams of alcohol Mary can drink per hour at a 3-hour party if she wants to stay below that blood alcohol level of 0.08? Begin with $a(0) = 0$.

TRYING DIFFERENT VALUES FOR $d$ ON YOUR CALCULATOR, YOU EVENTUALLY FIND $d \approx 17.577$ RESULTS IN $a(3) \approx 37$ grams

TO ONE DECIMAL PLACE

17.6 grams each hour

11. (10 points) The population of a town $t$ years from now is given by $P$. Suppose that the population of the town is currently 600.

(a) Determine a mathematical model which best describes this town's population if it grows at a continuous rate of 6.5% per year.

$$\frac{dP}{dt} = 0.065P$$

$$P(0) = 600$$

(b) Determine a mathematical model which best describes this town's population if it grows at an annual rate of 6.5% per year.

$$P(n) = 1.065P(n-1)$$

$$P(0) = 600$$

(c) Find an explicit formula for $P$ using the correct model from part (a).

$$P = 600e^{0.065t}$$

(d) Find an explicit formula for $P$ using the correct model from part (b).

$$P = 600(1.065)^n$$