

Name SOLUTIONS

- You must show sufficient work to justify your answers.

#1 (10 points) _____

#2 (10 points) _____

#3 (10 points) _____

#4 (15 points) _____

#5 (15 points) _____

#6 (15 points) _____

#7 (16 points) _____

#8 (9 points) _____

Total (100 points) _____

Test #1 _____ Test #2 _____ Test #3 _____ Total _____

If you skip the final exam, your course grade will be _____

1. (10 points) Find an explicit solution to the following initial value problem.

$$\frac{dh}{dv} = 0.25h, \quad h(0) = 200$$

$$h = 200e^{0.25v}$$

2. (10 points) Find an explicit solution to the following initial value problem.

$$\frac{dy}{dx} = 6xe^{-y} \quad y(1) = 0$$

$$\frac{dy}{dx} = \frac{6x}{e^y}$$

$$e^y dy = 6x dx$$

$$\int e^y dy = \int 6x dx$$

$$e^y = 3x^2 + C$$

$$y(1) = 0 \Rightarrow e^0 = 3(1)^2 + C \Rightarrow C = -2$$

$$e^y = 3x^2 - 2$$

$$y = \ln(3x^2 - 2)$$

3. (10 points) Find an explicit solution to the following initial value problem.

$$\frac{dq}{dt} = 3, \quad q(2) = 10$$

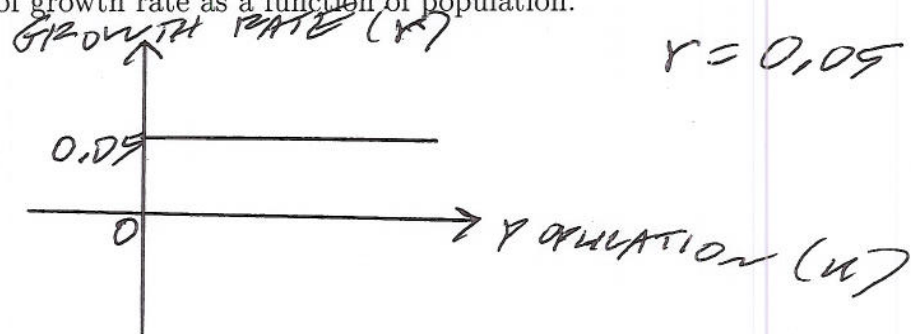
$$q = 3t + C$$

$$q(2) = 10 \Rightarrow 10 = 3(2) + C \Rightarrow C = 4$$

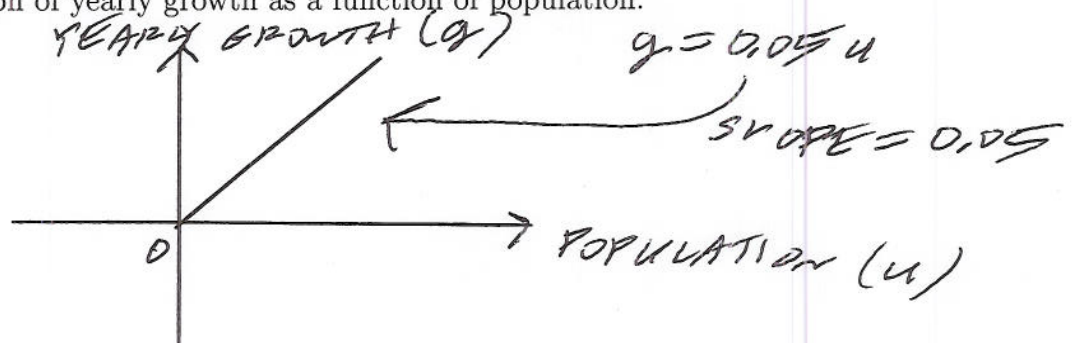
$$q = 3t + 4$$

4. (15 points) There are currently 100 rabbits, but the population is expected to grow exponentially by 5% each year from now on. Sketch the graphs described. I will mainly be looking at the shape of your graph along with the values of any intercepts being clearly marked.

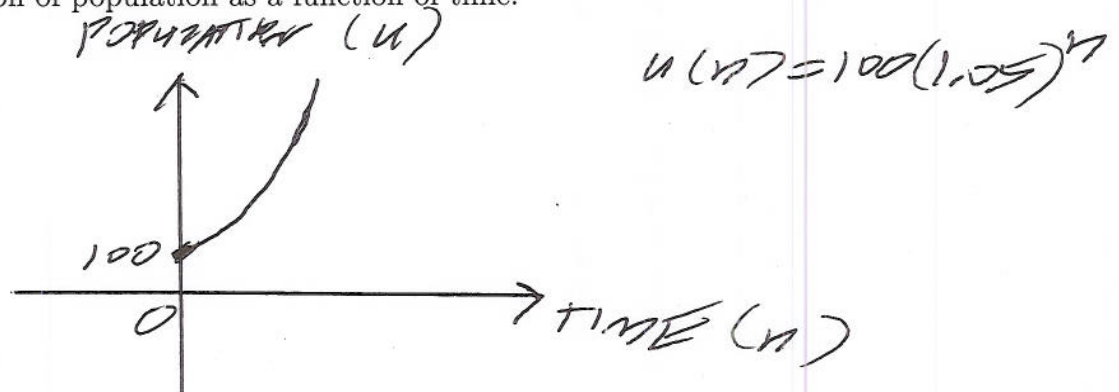
(a) Sketch a graph of growth rate as a function of population.



(b) Sketch a graph of yearly growth as a function of population.

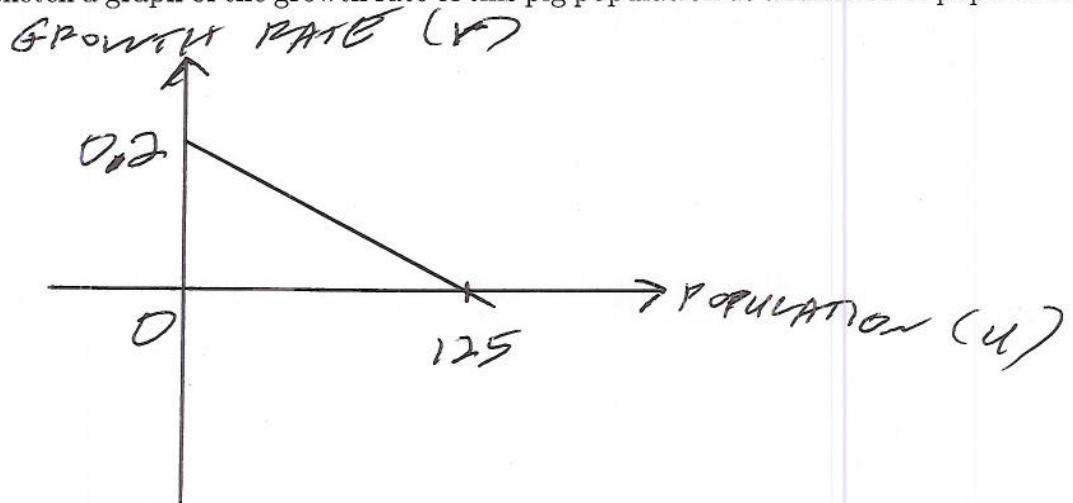


(c) Sketch a graph of population as a function of time.



5. (15 points) Suppose that a population of wild pigs grows logistically with an intrinsic growth rate of 20% and a carrying capacity of 125.

(a) Carefully sketch a graph of the growth rate of this pig population as a function of population.



(b) ~~Determine a discrete dynamical system to model this pig population.~~

$$u(n) = u(n-1) + 0.2u(n-1) \left(1 - \frac{u(n-1)}{125}\right)$$

(c) Determine the maximum interval of stability for this pig population.

$$(0, 750)$$

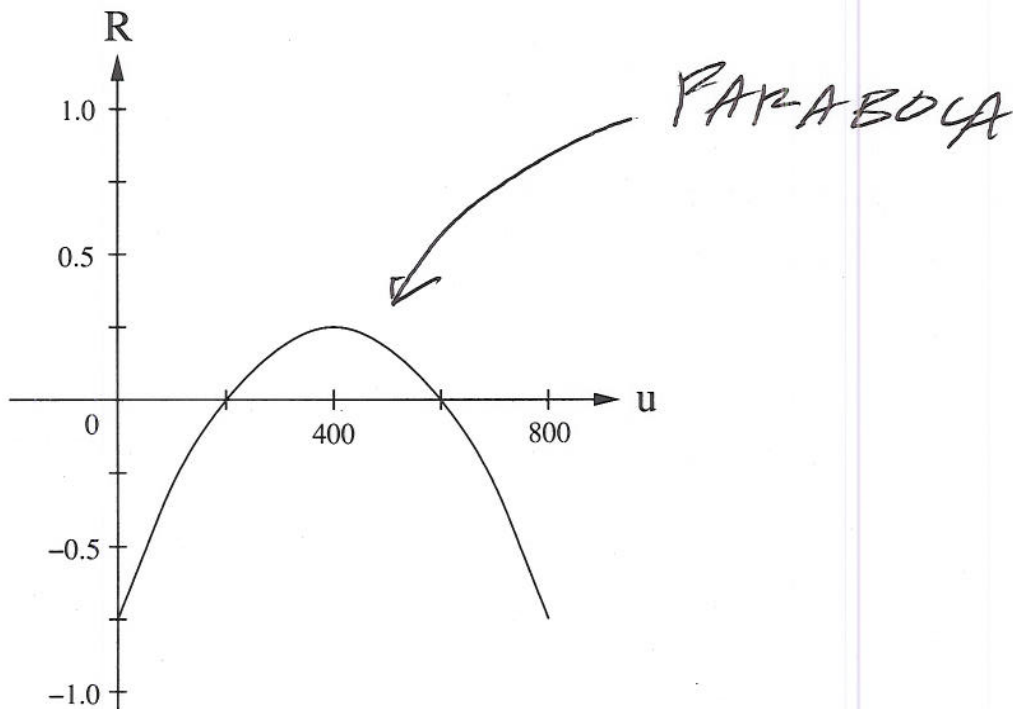
CAN USE DIFFERENT STARTING VALUES
ON CALCULATOR TO DETERMINE THIS
INTERVAL OR SIMPLY SOLVE

$$0 = u + 0.2u \left(1 - \frac{u}{125}\right)$$

6. (15 points) A population can be modeled by the following discrete dynamical system

$$u(n) = u(n-1) + R \cdot u(n-1)$$

where R is a function of the population u and is shown in the following graph.



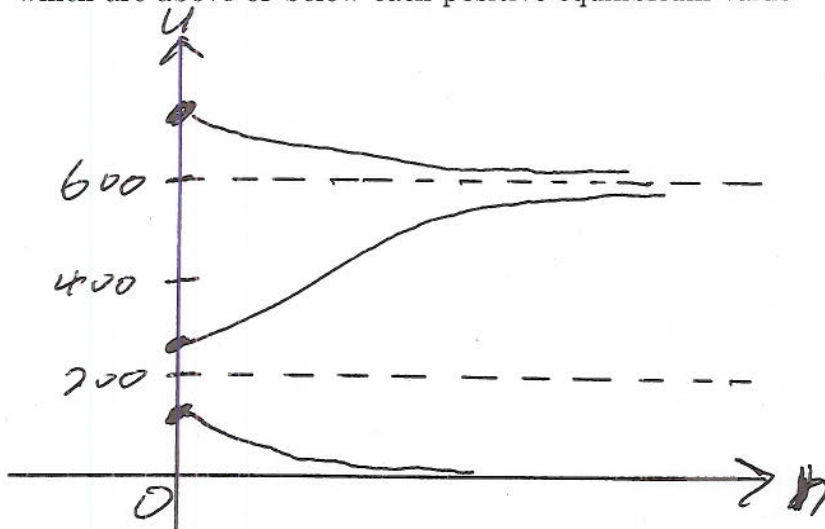
(a) Determine the intrinsic growth rate for this population?

25%

(b) Find all equilibrium values for this population.

0, 200, 600

- (c) Sketch a rough graph of the population as a function of time, being sure to show each equilibrium value clearly and being sure to show what happens to any initial populations which are above or below each positive equilibrium value.



- (d) Determine the minimum viable population.

200

- (e) If $u(0) = 150$, then what is the value of $u(6)$? *FORMULA FOR PARABOLA*

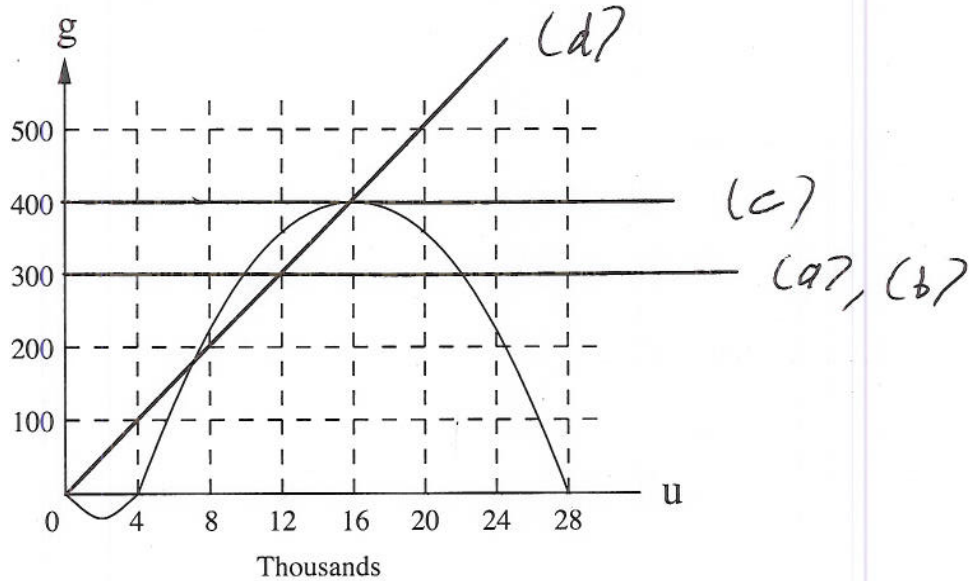
$$15 \quad R = \frac{-0.25}{200^2} (u - 400)^2 + 0.25$$

$$50 \quad u(n) = u(n-1) + \left(\frac{-0.25}{200^2} (u(n-1) - 400)^2 + 0.25 \right) \cdot u(n-1)$$

$$u(0) = 150$$

CALCULATOR GIVES $u(6) = 6.0589$

7. (16 points) The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram.



- (a) Estimate the stable equilibrium population if there is a constant yearly harvest of 300.

22000

- (b) Estimate the minimum viable population if there is a constant yearly harvest of 300.

10000

- (c) Estimate the maximum constant sustainable harvest and the equilibrium population size for this harvest.

400

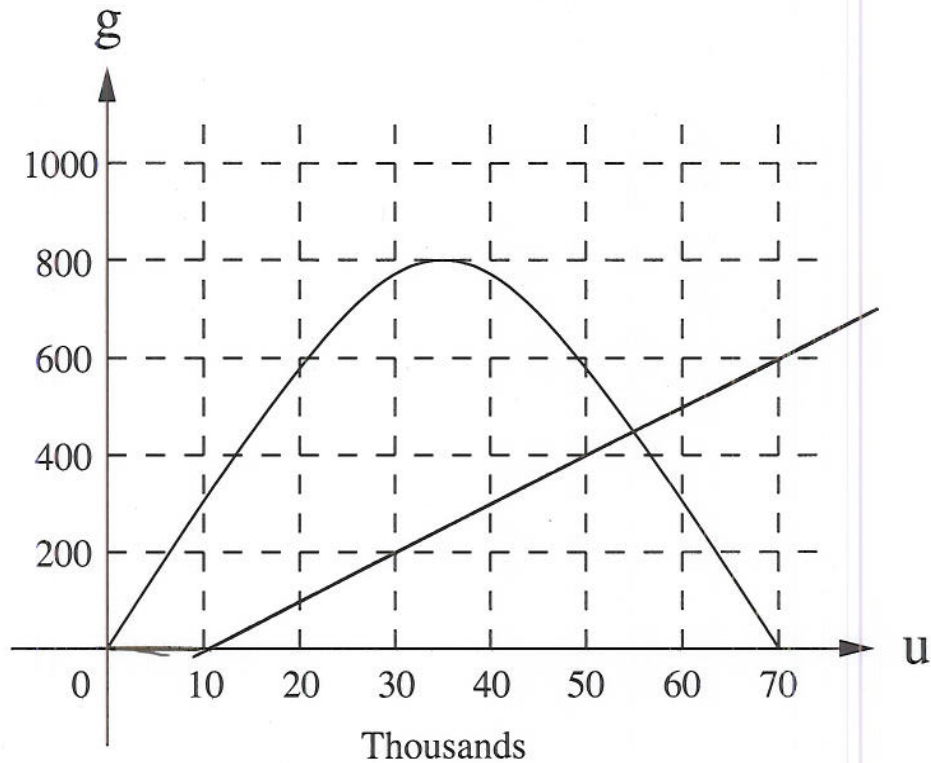
16000

- (d) Approximate the percent of the population that should be harvested each year to maximize the sustainable harvest.

$$h = \frac{400}{16000} u = 0.025 u$$

2.5%

8. (9 points) The graph of the function $g = ru$, which gives the growth of a species in a year in terms of the population size, is seen in the following diagram. Each year the harvesting strategy is to harvest 1% of the population in excess of 10,000.



- (a) Determine a formula for the yearly harvest, h , as a function of the population u .

$$h = \begin{cases} 0 & \text{if } u \leq 10000 \\ 0.01(u - 10000) & \text{if } u \geq 10000 \end{cases}$$

- (b) Estimate the stable equilibrium population.

55000

- (c) Estimate the eventual yearly harvest.

450