Math 172 **Quiz 4 Solutions** **Spring 2007**

1. The following dynamical system has a stable equilibrium point at $(u, v) = (20, 32)$. Given that $u(0) = 10$ and $v(0) = 20$, determine the rate at which $v(n)$ approaches equilibrium. Show all calculations you made to find the rate.

$$u(n) = 0.3u(n-1) - 0.5v(n-1) + 30$$
$$v(n) = 0.2u(n-1) + v(n-1) - 4$$

First we obtain a table of values for the functions u and v given our initial values.

n	u(n)	v(n)
0	10	20
1	23	18
2	27.9	18.6
3	29.07	20.18
4	28.631	21.994
5	27.5923	23.7202
6	26.41759	25.23866
7	25.305947	26.522178
8	24.3306951	27.5833674
9	23.50752483	28.44950642
10	22.827504239	29.151011386

n	u(n)	v(n)
11	22.2727455787	29.7165122338
12	21.8235675567	30.1710613495
13	21.4615395922	30.5357748609
14	21.1705744472	30.8280827793
15	20.9371309445	31.0621976688
16	20.750040449	31.2496238577
17	20.6002002059	31.3996319475
18	20.480244088	31.5196719886
19	20.3842372321	31.6157208062
20	20.3074107665	31.6925682527

Using the values for the function v shown in the table, we obtain the following:

$\frac{v(1)-32}{v(0)-32} \approx 1.16667$

$\frac{v(2)-32}{v(1)-32} \approx 0.95714$

$\frac{v(3)-32}{v(2)-32} \approx 0.88209$

$\frac{v(4)-32}{v(3)-32} \approx 0.84653$

$\frac{v(5)-32}{v(4)-32} \approx 0.82748$

$\frac{v(6)-32}{v(5)-32} \approx 0.81661$

$\frac{v(7)-32}{v(6)-32} \approx 0.81017$

$\frac{v(8)-32}{v(7)-32} \approx 0.80628$

$\frac{v(9)-32}{v(8)-32} \approx 0.80389$

$\frac{v(10)-32}{v(9)-32} \approx 0.80242$

$\frac{v(11)-32}{v(10)-32} \approx 0.80151$

$\frac{v(12)-32}{v(11)-32} \approx 0.80094$

$\frac{v(13)-32}{v(12)-32} \approx 0.80059$

$\frac{v(14)-32}{v(13)-32} \approx 0.80037$

$\frac{v(15)-32}{v(14)-32} \approx 0.80023$

$\frac{v(16)-32}{v(15)-32} \approx 0.80014$

$\frac{v(17)-32}{v(16)-32} \approx 0.80009$

$\frac{v(18)-32}{v(17)-32} \approx 0.80006$

$\frac{v(19)-32}{v(18)-32} \approx 0.80003$

$\frac{v(20)-32}{v(19)-32} \approx 0.80002$

It appears that $\lim\limits_{n\to\infty} \dfrac{v(n)-32}{v(n-1)-32} = 0.8$

So for large enough n, $v(n)$ is getting approximately 20% closer to its equilibrium value each day (assuming n is measured in days).

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2. For the following dynamical system, there is no equilibrium point, but the values for $u(n)$ (eventually) change by approximately the same amount.

$$u(n) = 0.9u(n-1) + 0.2v(n-1) + 600$$
$$v(n) = 0.1u(n-1) + 0.8v(n-1) + 400$$

(a) What is that approximate amount by which $u(n)$ eventually changes?

Since no initial values are given, you must first select values for $u(0)$ and $v(0)$. Regardless of the initial values chosen you should find that $\lim\limits_{n\to\infty} (u(n) - u(n-1)) \approx 666.7$

So for large enough n, $u(n)$ is increasing by approximately 666.7 each day (assuming n is measured in days).

(b) Does $u(n)$ appear to be more linear or exponential for large n?

For large enough n, $u(n)$ appears to be linear with an approximate slope of 666.7.

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3. Pregnant women metabolize some drugs at a slower rate than the rest of the population. The amount of caffeine in a pregnant woman's bloodstream decreases by approximately 7% each

hour (for others it decreases by 16%.) This is important because caffeine, like all psychoactive drugs, crosses the placenta to the fetus. Let $u(n)$ represent the amount of caffeine in a pregnant woman's bloodstream n hours after finishing a cup of coffee containing 100 mg of caffeine.

(a) Find a discrete dynamical system along with an initial value for $u(n)$.

$u(n) = 0.93u(n-1)$
$u(0) = 100$

(b) Find an explicit formula for $u(n)$.

$u(n) = 100(0.93)^n$

(c) Find the half-life of caffeine in a pregnant woman's bloodstream.

Half-life $= \dfrac{\ln(0.5)}{\ln(0.93)} \approx 9.6$ hours

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4. Find an explicit formula for this discrete dynamical system.

$u(n) = 0.8u(n-1) + 10$ and $u(0) = 70$

$u(n) = 20(0.8)^n + 50$
