Math 172 **Quiz 4** **February 16, 2007**

Name _____

1. The following dynamical system has a stable equilibrium point at $(u, v) = (20, 32)$. Given that $u(0) = 10$ and $v(0) = 20$, determine the rate at which $v(n)$ approaches equilibrium. Show all calculations you made to find the rate.

$$u(n) = 0.3u(n-1) - 0.5v(n-1) + 30$$
$$v(n) = 0.2u(n-1) + v(n-1) - 4$$

2. For the following dynamical system, there is no equilibrium point, but the values for $u(n)$ (eventually) change by approximately the same amount.

$$u(n) = 0.9u(n-1) + 0.2v(n-1) + 600$$
$$v(n) = 0.1u(n-1) + 0.8v(n-1) + 400$$

   (a) What is that approximate amount by which $u(n)$ eventually changes?

   (b) Does $u(n)$ appear to be more linear or exponential for large n?

3. Pregnant women metabolize some drugs at a slower rate than the rest of the population. The amount of caffeine in a pregnant woman's bloodstream decreases by approximately 7% each hour (for others it decreases by 16%.) This is important because caffeine, like all psychoactive drugs, crosses the placenta to the fetus. Let $u(n)$ represent the amount of caffeine in a pregnant woman's bloodstream n hours after finishing a cup of coffee containing 100 mg of caffeine.

   (a) Find a discrete dynamical system along with an initial value for $u(n)$.

   (b) Find an explicit formula for $u(n)$.

   (c) Find the half-life of caffeine in a pregnant woman's bloodstream.

4. Find an explicit formula for this discrete dynamical system.

   $u(n) = 0.8u(n-1) + 10$ and $u(0) = 70$