Math 172 **Quiz 5 Solutions** **Spring 2007**

1. Suppose a certain chemical is eliminated from the body by the kidneys and the liver. Let $u(n)$ represent the amount of this chemical in a person's bloodstream after n days. Assume that each day, the kidneys remove 20% of the chemical from the blood. Also assume that each day, the fraction of the chemical that is broken down by enzymes from the liver is given by

$$\frac{3}{4+u(n-1)}$$

Finally, assume that each day, the person takes a dose of 10 mg of this chemical. Develop a discrete dynamical system for $u(n)$. You do not need an initial value.

$$u(n) = u(n-1) - 0.2u(n-1) - \frac{3}{4+u(n-1)} \cdot u(n-1) + 10$$

$$u(n) = 0.8u(n-1) - \frac{3u(n-1)}{4+u(n-1)} + 10$$

2. Suppose the metabolism of some person is such that the discrete dynamical system modeling the elimination of alcohol is

$$a(n) = a(n-1) - \frac{10a(n-1)}{4+a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in the person's bloodstream after n hours of drinking d grams of alcohol per hour.

Suppose this person's weight is such that 40 grams of alcohol in the bloodstream represents a blood alcohol level of 0.08 (the amount in SC for a DWI conviction).

If this person drinks 3 cans of beer each hour for 3 hours, but then stops drinking for the next 3 hours, will his blood alcohol level fall below 40 grams? To justify your answer, you will need to fill in the remaining entries in the table below. Recall that each can of beer contains about 14 grams of alcohol.

Hint: Think carefully about the modifications needed to obtain the last 3 entries when he is no longer drinking.

n	a(n)
0	0
1	42.0
2	74.87
3	107.38
4	97.74
5	88.13
6	78.57

No, our model predicts that this person will still have nearly 79 grams of alchol in the bloodstream.

3. Suppose we have the following discrete dynamical system.

   $u(n) = 0.1u^2(n-1) + 0.3u(n-1) + 1$

   (a) Find each equilibrium value for this discrete dynamical system and state whether it appears to be stable or unstable.

   $E = 5$ (unstable), $E = 2$ (stable)

   (b) For each stable equilibrium value, determine the maximum interval of stability.

   For $E = 2$, the maximum interval of stability is $(-8, 5)$ (Note: for population models it may make more sense to say $(0, 5)$)

   (c) For each stable equilibrium value, suppose that $u(0)$ is within the maximum interval of stability. Approximate the rate at which $u(n)$ goes toward the equilibrium value.

   For $E = 2$ and an appropriate choice for $u(0)$ we obtain $\lim_{n\to\infty} \dfrac{u(n) - E}{u(n-1) - E} = 0.7$. Thus for large enough n, $u(n)$ is about 30% closer to equilibrium than $u(n-1)$.