

1. Given that $\frac{dP}{dt} = \sqrt[3]{P}$ and $P(0) = 100$, use Euler's Method with $\Delta t = 2$ to obtain an estimate for $P(10)$.

t	P	$\frac{dP}{dt} = P^{1/3}$	$P_{\text{new}} \approx P + \left(\frac{dP}{dt}\right) \cdot \Delta t$
0	100	4.64	$100 + 4.64(2)$
2	109.28	4.78	$109.28 + 4.78(2)$
4	118.85	4.92	$118.85 + 4.92(2)$
6	128.68	5.05	$128.68 + 5.05(2)$
8	138.78	5.18	$138.78 + 5.18(2)$
10	149.13		

$$P(10) \approx 149.1$$

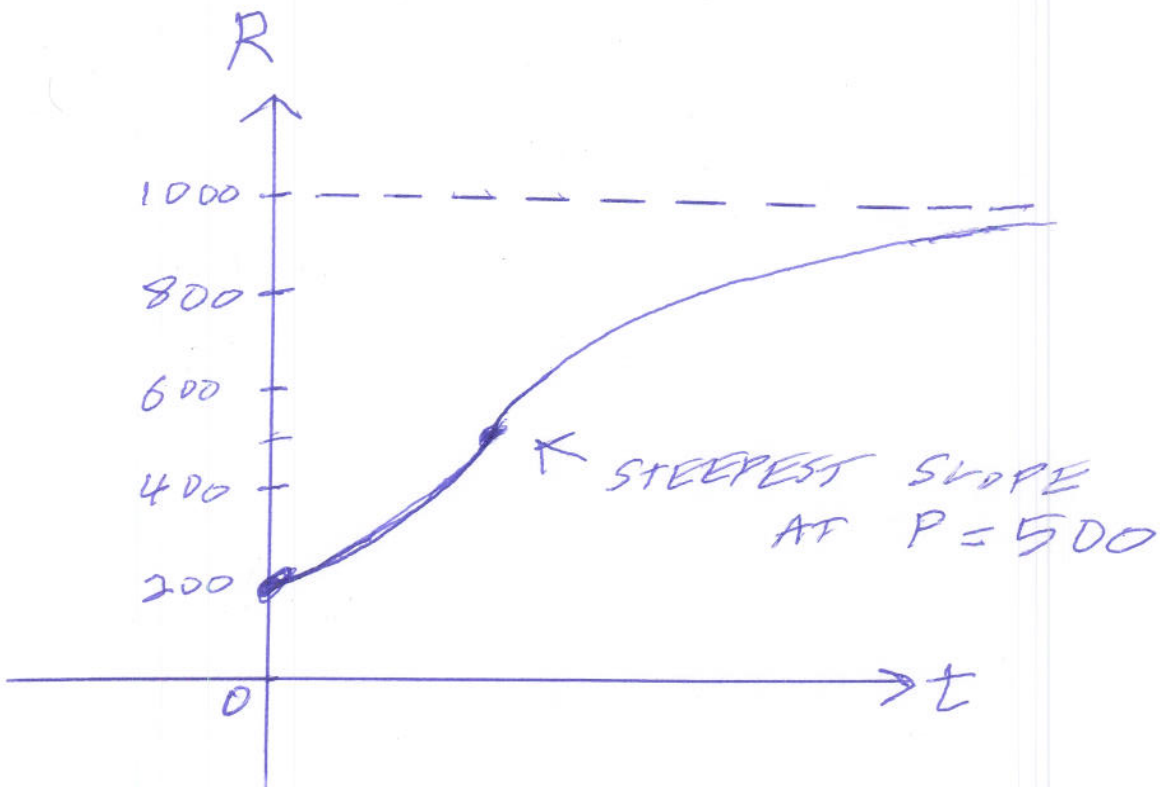
2. Suppose that 200 rabbits are released on Lady Tottington's estate, and that the population of these rabbits grows logistically with an intrinsic growth rate of 4% per month and a carrying capacity of 1000 rabbits.

- (a) Let R represent the number of rabbits on her estate t months after the initial release. Determine a differential equation with initial condition to model this population of rabbits.

$$\frac{dR}{dt} = 0.04R \left(1 - \frac{R}{1000} \right)$$

$$R(0) = 200$$

- (b) Sketch a plausible graph for this population of rabbits.



3. The population of a town is currently 600, but is projected to grow by 30 people per year from now on.

- (a) Determine a differential equation with initial condition to model this town's population. Use P for the population t years from now.

$$\frac{dP}{dt} = 30$$

$$P(0) = 600$$

- (b) Find an explicit formula for this town's population.

$$P = 30t + 600$$

4. The population of a town t years from now is given by P . Suppose that the population of the town is currently 400.

- (a) Determine a mathematical model which best describes this town's population if it grows at a continuous rate of 8.5% per year.

$$\frac{dP}{dt} = 0.085P$$

$$P(0) = 400$$

- (b) Determine a mathematical model which best describes this town's population if it grows at an annual rate of 8.5% per year.

$$\begin{aligned} P(n) &= P(n-1) + 0.085P(n-1) \\ &= ~~4~~ 1.085P(n-1) \end{aligned}$$

$$P(0) = 400$$

- (c) Find an explicit formula for P using each of your models above and use it to predict the population 10 years from now.

$$(a) \quad P(t) = 400e^{0.085t} \Rightarrow P(10) \approx 936$$

$$(b) \quad P(n) = 400(1.085)^n \Rightarrow P(10) \approx 904$$

5. Suppose that the population of a town is always growing at a rate which is proportional to the population itself. Suppose further that the population is currently 500 and is growing at a rate of 25 people per year. Find a differential equation with initial condition to model the population of this town. Use P for the population t years from now.

$$\frac{dP}{dt} = k \cdot P$$

$$25 = k \cdot 500$$

$$k = \frac{25}{500} = 0.05$$

$$\frac{dP}{dt} = 0.05P$$

$$P(0) = 500$$