- LINEAR GROWTH (m = slope, P_0 = initial value)

Type	Linear Model	Explicit Solution
continuous	$\dfrac{dP}{dt} = m$	$P = mt + P_0$
discrete	$P(n) = P(n-1) + m$	$P = mt + P_0$

- EXPONENTIAL GROWTH (r = growth rate, P_0 = initial value)

Type	Exponential Model	Explicit Solution
continuous	$\dfrac{dP}{dt} = rP$	$P = P_0 e^{rt}$
discrete	$P(n) = P(n-1) + rP(n-1)$	$P = P_0 (1+r)^n$

- LOGISTIC GROWTH (r = growth rate, k = carrying capacity, P_0 = initial value)

Type	Logistic Model
continuous	$\dfrac{dP}{dt} = rP\left(1 - \dfrac{P}{k}\right)$
discrete	$P(n) = P(n-1) + rP(n-1)\left(1 - \dfrac{P(n-1)}{k}\right)$

1. If P represents some population t years from now, and $\dfrac{dP}{dt} = 20$, then which of the following statements is correct?

   (a) $\boxed{P \text{ grows linearly by 20 people per year.}}$

   (b) P grows linearly by 120 people per year.

   (c) P grows exponentially by 20% per year.

   (d) P grows logistically with a carrying capacity of 20.

2. If P represents some population n years from now, and $P(n) = 1.2P(n-1)$, then which of the following statements is correct?

   (a) P grows linearly by 20 people per year.

   (b) P grows linearly by 120 people per year.

   (c) $\boxed{P \text{ grows exponentially by 20\% per year.}}$

   (d) P grows logistically with a carrying capacity of 120.

3. The population of a city was 5000 in 1980. Since then the population has been increasing by 100 people per year.

   (a) Determine a discrete dynamical system with initial value to model the city's population.

   $\boxed{P(n) = P(n-1) + 100 \qquad \text{and} \qquad P(0) = 5000}$

   (b) Determine a differential equation with initial value to model the city's population.

   $\boxed{\frac{dP}{dt} = 100 \qquad \text{and} \qquad P(0) = 5000}$

   (c) Determine an explicit formula for the city's population.

   $\boxed{P = 100t + 5000}$

   (d) What does your model predict for the city's population in the year 2000?

   $\boxed{7000}$

   (e) When does your model predict the population will have reached 10000?

   $\boxed{\text{In the year 2030}}$

4. An initial deposit of \$200 is made into an account with an annual percentage rate (APR) of 3% compounded annually.

   (a) Determine a discrete dynamical system with initial value to model the amount of money in this account.

   $\boxed{A(n) = 1.03A(n-1) \qquad \text{and} \qquad A(0) = 200}$

   (b) Determine an explicit formula for the amount of money in this account.

   $\boxed{A = 200(1.03)^n}$

   (c) How much money will the account hold 8 years after the initial deposit?

   $\boxed{\$253.35}$

(d) How long will it take until the balance in this account is \$500?

$\boxed{31.0 \text{ years}}$

5. An initial deposit of 200 is made into an account with an annual percentage rate (APR) of 3% compounded continuously.

   (a) Determine a differential equation with initial value to model the amount of money in this account.

   $\boxed{\frac{dA}{dt} = 0.03A \qquad \text{and} \qquad A(0) = 200}$

   (b) Determine an explicit formula for the amount of money in this account.

   $\boxed{A = 200e^{0.03t}}$

   (c) How much money will the account hold 8 years after the initial deposit?

   $\boxed{\$254.25}$

   (d) How long will it take until the balance in this account is \$500?

   $\boxed{30.5 \text{ years}}$

6. Why did I suggest using a discrete dynamical system for problem (4), but a differential equation for problem (5)?

   The interest was compounded annually for problem (4) but continuously for problem (5)

7. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.

   (a) Determine a discrete dynamical system with initial value to model the number of fish in this pond.

   If we let $F(n)$ represent the number of fish n years after 1970, then

   $\boxed{F(n) = 1.05F(n-1) \qquad \text{and} \qquad F(0) = 300}$

   (b) Enter this system into your calculator to make a table of values for the number of fish in the pond each year from 1970 to 1980.

n	$F(n)$
0	300.00
1	315.00
2	330.75
3	347.29
4	364.65
5	382.88
6	402.03
7	422.13
8	443.24
9	465.40
10	488.67

   (c) Determine an explicit formula for the number of fish in the pond.

   $\boxed{F(n) = 300(1.05)^n}$

(d) When will the fish population reach 750?

$\boxed{\text{18.8 years later}}$

8. The number of fish in a pond was 300 in 1970. Since then the number of fish has been increasing by 5% per year.

(a) Determine a differential equation with initial value to model the number of fish in this pond.

$\boxed{\frac{dF}{dt} = 0.05F \qquad \text{and} \qquad F(0) = 300}$

(b) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of fish in the pond each year from 1970 to 1980.

Since $\Delta t = 1$, our table will have the same values as the discrete dynamical system shown in #7b.

(c) Determine an explicit formula for the number of fish in the pond.

$\boxed{F(t) = 300e^{0.05t}}$

(d) When will the fish population reach 750?

$\boxed{\text{18.3 years later}}$

9. Which model was best to use for the fish population – the discrete dynamical system or the differential equation? Why?

$\boxed{\text{discussed in class}}$

10. There are currently 5000 deer in a forest. Suppose the population of deer grows logistically with an intrinsic growth rate of 6% and a carrying capacity of 20,000.

(a) Sketch a rough graph of the deer population.

$\boxed{\text{graph shown in class}}$

(b) Determine a discrete dynamical system with initial value to model the deer population.

If we let $D(n)$ represent the number of deer n years from now, then

$$\boxed{D(n) = D(n-1) + 0.06D(n-1)\left(1 - \frac{D(n-1)}{20000}\right) \qquad \text{and} \qquad D(0) = 5000}$$

(c) Make a table of values for the number of deer your discrete model predicts for the next 4 years.

n	$D(n)$
0	5000.0
1	5225.0
2	5456.6
3	5694.7
4	5939.1

(d) Determine a differential equation with initial value to model the deer population.

$$\boxed{\frac{dD}{dt} = 0.06D\left(1 - \frac{D}{20000}\right) \qquad \text{and} \qquad D(0) = 5000}$$

(e) Use Euler's Method with $\Delta t = 1$ to make a table of values for the number of deer your continuous model predicts for the next 4 years.

t_{old}	D_{old}	D'_{old}	$D_{new} \approx D_{old} + D'_{old} \cdot \Delta t$
0	5000.0	225.0	5225.0
1	5225.0	231.6	5456.6
2	5456.6	238.1	5694.7
3	5694.7	244.4	5939.1
4	5939.1		

Note that since $\Delta t = 1$, our table has the same values as the discrete dynamical system shown in part c.

(f) Use Euler's Method with $\Delta t = 0.5$ to make a table of values for the number of deer your continous model predicts for the next 4 years.

t_{old}	D_{old}	D'_{old}	$D_{new} \approx D_{old} + D'_{old} \cdot \Delta t$
0.0	5000.00	225.00	5112.50
0.5	5112.50	228.34	5226.67
1.0	5226.67	231.65	5342.50
1.5	5342.50	234.92	5459.96
2.0	5459.96	238.16	5579.04
2.5	5579.04	241.37	5699.73
3.0	5699.73	244.52	5821.99
3.5	5821.99	247.63	5945.81
4.0	5945.81		

(g) How close are the approximations for discrete and continuous models?

discussed in class