

1. Consider the population model

$$\frac{dP}{dt} = 0.3P \left(1 - \frac{P}{200}\right) \left(\frac{P}{50} - 1\right)$$

where P is the population at time t .

(a) For which values of P is the population increasing?

$$50 < P < 200$$

MATHEMATICALLY
(ALSO FOR $P < 0$
BUT WE CAN
IGNORE NEGATIVE
VALUES HERE

(b) For which values of P is the population decreasing?

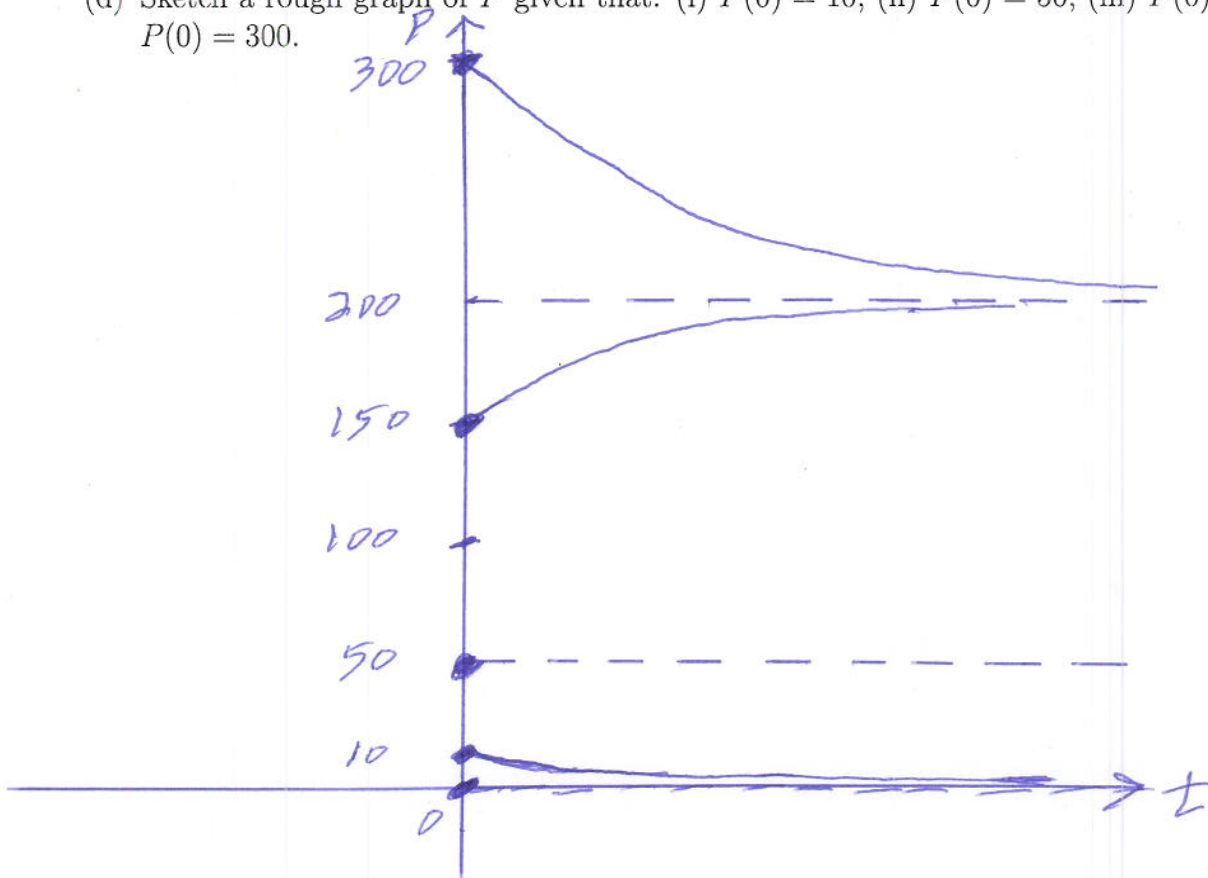
$$P > 200 \text{ OR } 0 < P < 50$$

(c) For which values of P is the population in equilibrium? Determine whether each of these equilibrium values is stable or unstable.

$P = 0$ AND $P = 200$ ARE STABLE EQ. VALUES

$P = 50$ IS AN UNSTABLE EQ. VALUE

(d) Sketch a rough graph of P given that: (i) $P(0) = 10$, (ii) $P(0) = 50$, (iii) $P(0) = 150$, (iv) $P(0) = 300$.



2. Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.5(P-4)(P+3)(P^2-25)$$

$$0 = 0.5(P-4)(P+3)(P-5)(P+5)$$

EQUILIBRIUM VALUES ARE

$$P = 4, -3, 5, \text{ AND } -5$$

3. Suppose y is a function of t which satisfies the differential equation below.

$$\frac{dy}{dt} = 0.25(y-10)(20-y)$$

Sketch plausible graphs for y as a function of t given each initial value below. Your graphs should clearly show if the y -values approach any particular values (i.e. horizontal asymptotes). You should draw all five graphs together on one set of coordinate axes.

(a) $y(0) = 25$

(b) $y(0) = 20$

(c) $y(0) = 15$

(d) $y(0) = 10$

(e) $y(0) = 5$

