Math 172 **Solutions for Calculus Review (Derivatives)** **Spring 2008**

1. (a) $\dfrac{dh}{dt} = 15t^2 + \dfrac{2}{3}t + 7$

   (b) $P'(t) = 0$

   (c) $g'(r) = \dfrac{1}{r} + \dfrac{1}{2\sqrt{r}}$

   (d) $h'(x) = \dfrac{-40}{x^6}$

   (e) $W'(t) = \dfrac{-2}{t^5}$

   (f) $\dfrac{dy}{dx} = -2x^{-3/2}$

   (g) $\dfrac{dw}{dx} = \dfrac{-1}{3}x^{-4/3}$

   (h) $\dfrac{dh}{dt} = 1 - \dfrac{1}{t^2}$

   (i) $\dfrac{dP}{dt} = 200e^{2t}$

   (j) $\dfrac{dy}{dx} = \dfrac{4}{x}$

   (k) $\dfrac{dw}{dx} = 2xe^x + x^2e^x$

   (l) $f'(x) = 99(x^2+1)^{98}(2x)$

   (m) $f'(x) = \dfrac{-10}{(x+3)^{11}}$

   (n) $u'(n) = 50e^{0.5n}$

   (o) $f'(x) = -20xe^{5-x^2}$

   (p) $v'(n) = \dfrac{-10}{e^n}$

   (q) $f'(t) = 3000\ln(1.02)(1.02)^t$

   (r) $\dfrac{dx}{dt} = \dfrac{1}{2}\left(t^3+1\right)^{-1/2}\left(3t^2\right)$

   (s) $\dfrac{dy}{dx} = e^{\sqrt{x}}\left(\dfrac{1}{2}x^{-1/2}\right)$

(t) $\dfrac{dw}{dt} = -30e^{-0.6t}$

(u) $\dfrac{dy}{dx} = 3x^2 + 10 - 2x^{-3}$

(v) $\dfrac{dh}{dx} = \dfrac{2x\left(x^5 + 10x^3 + 1\right) - x^2\left(5x^4 + 30x^2\right)}{\left(x^5 + 10x^3 + 1\right)^2}$

(w) $\dfrac{dW}{dt} = \dfrac{t^3e^t - 3t^2e^t}{t^6}$

(x) $f'(x) = 2xe^{-1.5x} - 1.5x^2e^{-1.5x}$

(y) $\dfrac{dy}{dx} = \dfrac{1}{2}\left(\ln\left(x^3 + 2\right)\right)^{-1/2} \cdot \dfrac{3x^2}{x^3 + 2}$

(z) $f'(x) = \dfrac{1}{2x}$

2. Since $P(t) = 900e^{-0.05t}$, we find $P'(t) = -45e^{-0.05t}$. In 1980 we obtain $P(30) \approx 201$ and $P'(30) \approx -10$. So in 1980 the town has a population of 201 and is decreasing by 10 people per year.

3. Since $f(x) = x^2 - 4$, we find $f'(x) = 2x$. Since $f(3) = 5$ and $f'(3) = 6$, we need to find a line which goes through the point $(3, 5)$ and has slope 6. The answer is $y = 6x - 13$.

4. Since $P(t) = 10e^{-t}$, we find $P'(t) = -10e^{-t}$. Since $P(0) = 10$ and $P'(0) = -10$, we need to find a line which goes through the point $(0, 10)$ and has slope -10. The answer is $P = -10t + 10$.

5. Since $h(t) = 63(0.91)^t$, we find $h(t) = 63\ln(0.91)(0.91)^t$. At time $t = 11$ we have $h(11) \approx 22.3$ and $h'(11) \approx -2.1$. So at that time the witch was 22.3 inches and was melting at 2.1 inches per minute.

6. (a) $f(-2)$ has the largest positive value. That is, the largest positive y-value on the graph occurs at $x = -2$.

   (b) $f'(2)$ has the largest positive value. That is, the largest positive slope on the graph occurs at $x = 2$.

7. The derivative is positive at the points labelled A and E. The derivative is negative at the points labelled B, C, and D.

8. We need to use the functions $h(t) = -16t^2 + 96t + 160$ and $h'(t) = -32t + 96$.

   (a) $\dfrac{h(2) - h(0)}{2} = 64$ feet per second

   (b) $h'(1.5) = 48$ feet per second

   (c) Max. height at 3 seconds (when $h'(t) = 0$)

   (d) Max. height is 304 feet

   (e) Ball hits ground at 7.36 seconds

   (f) Velocity is -139.48 feet per second

9. We need to use the functions $f(t) = 90 - 54e^{-0.2t}$ and $f'(t) = 10.8e^{-0.2t}$.

   (a) $f(0) = 36$ degrees

   (b) $f'(0) = 10.8$ degrees per minute

   (c) $f'(6) \approx 3.25$ degrees per minute

   (d) 9.5477 or almost 10 minutes

10. (a) If $f(6) \approx 180$, then six days after the student returned to campus, there were a total of 180 students infected with the flu virus.

    (b) If $f'(6) \approx 10$, then six days after the student returned to campus, the number of infected students was increasing by 10 students per day.

    (c) 190 students

11. (a) When my cat is 13 years old, she weighs 14 pounds and is gaining 0.25 pounds per year.

    (b) If she continues to gain 0.25 pounds per year, then I expect her to weight 14.5 pounds at the age of 15.