Math 172 (Section 1) Test 1 Spring 2008

Name SOLUTIONS

Seat # _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your Student ID when you turn in your test.

Do not write below this line

#1 (8 points) _____	#7 (8 points) _____
#2 (8 points) _____	#8 (8 points) _____
#3 (8 points) _____	#9 (8 points) _____
#4 (8 points) _____	#10 (8 points) _____
#5 (8 points) _____	#11 (12 points) _____
#6 (8 points) _____	#12 (8 points) _____
	Total (100 points) _____

1. (8 points) Consider the following discrete dynamical system.

$$P(t+1) = 1.2P(t) - 40$$

If $P(0) = 300$, then what is the value of $P(4)$?

t	$P(t)$
0	300
1	320
2	344
3	372.8
4	407.36

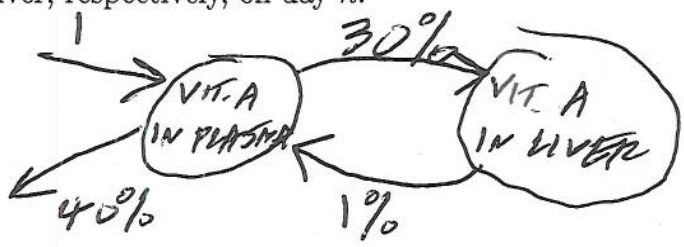
$P(4) = 407.36$

2. (8 points) Smoking one pack of cigarettes a day will cause you to absorb about 2.7 mg of the heavy metal pollutant cadmium each year. Suppose that the cigarettes are the only source of cadmium in your body, and that your body eliminates 7% of its cadmium each year. Develop a discrete dynamical system (without an initial value) to represent $C(t)$, the number of milligrams of cadmium in your body t years after starting to smoke one pack of cigarettes a day.

$C(t) = C(t-1) - 0.07C(t-1) + 2.7$

$C(t) = 0.93C(t-1) + 2.7$

3. (8 points) Vitamin A is stored primarily in our plasma and our liver. Suppose that 40% of the vitamin A in the plasma is filtered out by the kidneys each day and that 30% of the vitamin A in the plasma is absorbed into the liver each day. Also assume that 1% of the vitamin A in the liver is absorbed back into the plasma each day. Suppose you have a daily intake of 1 mg of vitamin A each day, which goes directly into the plasma. Develop a discrete dynamical system (without initial values) to represent $P(n)$ and $L(n)$, the number of milligrams of vitamin A in the plasma and liver, respectively, on day n.



$P(n) = P(n-1) + 0.01L(n-1) + 1 - 0.4P(n-1) - 0.3P(n-1)$

$L(n) = L(n-1) + 0.3P(n-1) - 0.01L(n-1)$

$$P(n) = 0.3P(n-1) + 0.01L(n-1) + 1$$
$$L(n) = 0.3P(n-1) + 0.99L(n-1)$$

4. (8 points) Suppose that $u(n)$ and $v(n)$ represent the number of milligrams of drugs U and V, respectively, in the bloodstream n days after an initial dose of each. The discrete dynamical system for this is shown below.

n	$u(n)$	$v(n)$
0	32	9
1	48.5	22.7
2	54.44	32.43
3	54.935	38.304
4	53.27	40.802
5	51.841	41.362

$$u(n) = 0.6u(n-1) - 0.3v(n-1) + 32$$
$$v(n) = 0.3u(n-1) + 0.4v(n-1) + 9$$
$$u(0) = 32$$
$$v(0) = 9$$

(a) How many milligrams of each drug are in the bloodstream 5 days after the initial dose?

$u(5) = 51.84125$ mg $\quad v(5) = 41.36178$ mg

(b) Assuming that the patient is to continue taking this drug for a long time, what was the doctor's **target goal** for the desired amount of each drug in the bloodstream?

n	$u(n)$	$v(n)$
0	32	9
5	51.841	41.362
10	49.876	39.978
15	50.006	39.997
20	50.000	40.000
25	50.000	40.000

TARGET GOAL FOR u IS 50 mg

TARGET GOAL FOR v IS 40 mg

5. (8 points) Suppose $P(t)$ represents some population t years from now, and that this population is modeled by the following discrete dynamical system.

$$P(t) = P(t-1) + 30$$
$$P(0) = 60$$

Which one of the following statements follows from this model?

(a) The population will increase by 30% per year.
(b) The population will increase by 30 people per year.
(c) The population will increase by 40% per year.
(d) The population will increase by 40 people per year.
(e) The population will increase by 60% per year.
(f) The population will increase by 60 people per year.
(g) The population will increase by 70% per year.
(h) The population will increase by 70 people per year.

6. (8 points) Suppose $a(n)$ represents the number of milligrams of some drug in the bloodstream n hours from now, and that the amount of this drug in the bloodstream is modeled by the following discrete dynamical system.

$$a(n) = 0.65a(n-1)$$
$$a(0) = 50$$

Which one of the following statements follows from this model?

(a) The amount of drug in the bloodstream will increase by 35% per hour.
(b) The amount of drug in the bloodstream will decrease by 35% per hour.
(c) The amount of drug in the bloodstream will increase by 35 milligrams per hour.
(d) The amount of drug in the bloodstream will decrease by 35 milligrams per hour.
(e) The amount of drug in the bloodstream will increase by 50% per hour.
(f) The amount of drug in the bloodstream will decrease by 50% per hour.
(g) The amount of drug in the bloodstream will increase by 50 milligrams per hour.
(h) The amount of drug in the bloodstream will decrease by 50 milligrams per hour.
(i) The amount of drug in the bloodstream will increase by 65% per hour.
(j) The amount of drug in the bloodstream will decrease by 65% per hour.
(k) The amount of drug in the bloodstream will increase by 65 milligrams per hour.
(l) The amount of drug in the bloodstream will decrease by 65 milligrams per hour.

7. (8 points) The equilibrium point is $(u^*, v^*) = (22.5, -1.25)$ for the following discrete dynamical system. Determine whether or not this equilibrium is stable. Show all necessary work to justify your answer.

$$u(n) = 0.8u(n-1) + 0.4v(n-1) + 5$$
$$v(n) = -0.3u(n-1) + 0.4v(n-1) + 6$$

n	u(n)	v(n)
0	10	-5
4	20.272	1.912
8	23.132	-1.342
12	22.537	-1.362
16	22.474	-1.238
20	22.501	-1.247
24	22.501	-1.251

n	u(n)	v(n)
0	35	5
4	25.4	-4.84
8	21.818	-1.22
12	22.441	-1.118
16	22.53	-1.261
20	22.500	-1.254
24	22.499	-1.249

n	u(n)	v(n)
0	15	4
4	23.174	-0.6368
8	22.731	-1.492
12	22.458	-1.255
16	22.494	-1.241
20	22.502	-1.25
24	22.5	-1.25

n	u(n)	v(n)
0	30	-8
4	21.418	-1.606
8	22.299	-0.9708
12	22.558	-1.257
16	22.502	-1.26
20	22.498	-1.249
24	22.5	-1.25

STABLE

8. (8 points) A discrete dynamical system for $u(n)$ has an equilibrium value of 50. With an initial value of $u(0) = 60$, we have the following table of values for $u(n)$.

n	u(n)
0	60
1	56
2	53.6
3	52.16
4	51.296
5	50.778
6	50.467
7	50.28
8	50.168
9	50.101

$\frac{u(1)-50}{u(0)-50} = \frac{56-50}{60-50} = 0.6$

$\frac{u(2)-50}{u(1)-50} = \frac{53.6-50}{56-50} = 0.6$

$\frac{u(3)-50}{u(2)-50} = \frac{52.16-50}{53.6-50} = 0.6$

From this data, which one of the following statements is true?

(a) u is getting 25% closer to equilibrium each time period.

(b) u is getting 30% closer to equilibrium each time period.

(c) u is getting 35% closer to equilibrium each time period.

(d) u is getting 40% closer to equilibrium each time period.

(e) u is getting 45% closer to equilibrium each time period.

(f) u is getting 50% closer to equilibrium each time period.

(g) u is getting 55% closer to equilibrium each time period.

(h) u is getting 60% closer to equilibrium each time period.

(i) u is getting 65% closer to equilibrium each time period.

(j) u is getting 70% closer to equilibrium each time period.

(k) u is getting 75% closer to equilibrium each time period.

(l) u is getting 80% closer to equilibrium each time period.

$\frac{u(8)-50}{u(7)-50} \approx 0.6$

$\frac{u(9)-50}{u(8)-50} \approx 0.6$

9. (8 points) Compute the equilibrium point (p^*, q^*) for the following discrete dynamical system.

$$p(t+1) = 2p(t) - 0.8q(t) + 1$$
$$q(t+1) = 0.5p(t) + 0.5q(t) + 3$$

$p^* = 2p^* - 0.8q^* + 1$

$q^* = 0.5p^* + 0.5q^* + 3$

$-p^* + 0.8q^* = 1$

$-0.5p^* + 0.5q^* = 3$

$-p^* + 0.8q^* = 1$

$p^* - q^* = -6$

$-0.2q^* = -5$

$q^* = \frac{-5}{-0.2} = 25$

$p^* - 25 = -6$

$p^* = 19$

$(p^*, q^*) = (19, 25)$

10. (8 points) We are given the following discrete dynamical system.

IF WE START WITH $u(0) = 0$ AND $v(0) = 0$,

$$u(n) = 0.7u(n-1) + 0.1v(n-1) + 50$$
$$v(n) = 0.3u(n-1) + 0.9v(n-1) + 80$$

THEN Although the function $u(n)$ is not linear, as n increases the function starts to look like a line with slope 32.5.

$u(1) - u(0) = 50$

$u(2) - u(1) = 43$

$\vdots$

$u(10) - u(9) = 32.67636$

$u(20) - u(19) = 32.50107$

$u(30) - u(29) = 32.50001$

$\vdots$

11. (12 points) Find an explicit formula for an expression which satisfies each of the following discrete dynamical systems.

(a) $P(n+1) = P(n) - 8$ and $P(0) = 120$

$P(n) = -8n + 120$

(b) $u(n) = 1.1u(n-1)$ and $u(0) = 50$

$u(n) = 50(1.1)^n$

(c) $u(n) = 1.25u(n-1) - 6$ and $u(0) = 30$

$u^* = 1.25u^* - 6$

$-0.25u^* = -6$

$u^* = \frac{-6}{-0.25} = 24$

$u(n) = K(1.25)^n + 24$

$u(0) = 30$ 50

$u(n) = 6(1.25)^n + 24$

12. (8 points) Tom makes a single \$1200 purchase on his credit card and then never uses it again. The card carries an 19.2% annual interest rate compounded monthly, and he only pays back his minimum monthly payment of \$25 per month.

(a) Write down a discrete dynamical system with initial condition to represent the amount Tom owes after n monthly payments.

$b(n) = b(n-1) + \frac{0.192}{12} b(n-1) - 25$

$b(n) = 1.016\, b(n-1) - 25$

$b(0) = 1200$

(b) How many months will it take Tom to pay off this credit card debt?

n	$b(n)$
0	1200
⋮	
92	1.0462
93	-23.94

93 MONTHS

FIRST 92 PAYMENTS ARE FOR \$25.

THE 93rd PAYMENT IS FOR $25 - 23.94 = \$1.06$