GREEN TEST

Math 172 (Section 1) Test 2 Spring 2008

Name SOLUTIONS

Seat # _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your Student ID when you turn in your test.

Do not write below this line

#1 (5 points)	_____	#8d (5 points)	_____
#2 (5 points)	_____	#8e (5 points)	_____
#3 (5 points)	_____	#8f (5 points)	_____
#4 (5 points)	_____	#9 (5 points)	_____
#5 (5 points)	_____	#10 (5 points)	_____
#6 (5 points)	_____	#11a (5 points)	_____
#7 (10 points)	_____	#11b (5 points)	_____
#8a (5 points)	_____	#11c (5 points)	_____
#8b (5 points)	_____	#11d (5 points)	_____
#8c (5 points)	_____		

TOTAL (100 points) _____

1. (5 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 40$ and $\frac{dP}{dt} = 0.1P$?

   (a) P grows linearly by 10 people per year.

   (b) P grows linearly by 40 people per year.

   (c) P grows linearly by 60 people per year.

   (d) P grows linearly by 90 people per year.

   (e) P grows exponentially with a continuous growth rate of 10% per year.

   (f) P grows exponentially with a continuous growth rate of 40% per year.

   (g) P grows exponentially with a continuous growth rate of 60% per year.

   (h) P grows exponentially with a continuous growth rate of 90% per year.

   (i) P grows logistically with a carrying capacity of 10.

   (j) P grows logistically with a carrying capacity of 40.

   (k) P grows logistically with a carrying capacity of 60.

   (l) P grows logistically with a carrying capacity of 90.

2. (5 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 40$ and $\frac{dP}{dt} = 10$?

   (a) P grows linearly by 10 people per year.

   (b) P grows linearly by 40 people per year.

   (c) P grows linearly by 60 people per year.

   (d) P grows linearly by 90 people per year.

   (e) P grows exponentially with a continuous growth rate of 10% per year.

   (f) P grows exponentially with a continuous growth rate of 40% per year.

   (g) P grows exponentially with a continuous growth rate of 70% per year.

   (h) P grows exponentially with a continuous growth rate of 80% per year.

   (i) P grows logistically with a carrying capacity of 10.

   (j) P grows logistically with a carrying capacity of 40.

   (k) P grows logistically with a carrying capacity of 70.

   (l) P grows logistically with a carrying capacity of 80.

3. (5 points) It has been raining heavily since midnight. Let $f(t)$ represent the total number of inches of rain which have fallen in the t hours since midnight. Given that $f'(2) = 4$, which one of the following sentences must be true?

(a) From midnight to 2:00AM it rained a total of 4 inches.

(b) From midnight to 4:00AM it rained a total of 2 inches.

(c) From midnight to 2:00AM it was raining at an average rate of 4 inches per hour.

(d) From midnight to 4:00AM it was raining at an average rate of 2 inches per hour.

(e) At 2:00AM it was raining at a rate of 4 inches per hour.

(f) At 4:00AM it was raining at a rate of 2 inches per hour.

4. (5 points) The maximum range of a projectile is proportional to the square of its velocity. A baseball pitcher can throw a ball at 60 miles per hour with a maximum range of 242 feet. What would his maximum range be if he could throw the ball at 65 miles per hour?

$R = kv^2$

$242 = k(60)^2$

$k = \frac{242}{3600} = 0.06\overline{72}$

$R = 0.06\overline{72}v^2$

$R = 0.06\overline{72}(65)^2 \approx 284.0$ feet

5. (5 points) A model for the population of a town predicts the population t years from now to be given by $P(t) = 400e^{-0.02t}$. How quickly in people per year is the population predicted to be changing 25 years from now?

$P'(t) = 400e^{-0.02t} \cdot (-0.02)$

$= -8e^{-0.02t}$

$P'(25) = -8e^{-0.02(25)}$

$= -8e^{-0.5} \approx -4.85$ people/year

DECREASING BY 4.85 people/year

6. (5 points) Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.6(P^2 - 4)(P^2 + 9)(P - 15)$$

$= 0.6(P-2)(P+2)(P^2+9)(P-15)$

$P^* = 2, -2,$ OR 15

7. (10 points) Suppose P is a function of t whose growth is determined by the differential equation with initial condition shown. Use Euler's Method with $\Delta t = 3$ to approximate $P(9)$. Each step in your calculation should be correctly rounded off to three places after the decimal point.

$$\frac{dP}{dt} = \frac{8}{P^2} \quad \text{and} \quad P(0) = 5$$

t	P	P'	P_{next}
0	5	0.32	$5 + 0.32(3) = 5.96$
3	5.96	0.225	$5.96 + 0.225(3) = 6.635$
6	6.635	0.182	$6.635 + 0.182(3) = 7.181$
9	7.181		

$P(9) \approx 7.181$

≈ 7.2

8. (5 points each) Let P represent a town's population t years from now. Give a differential equation which models the population under the following conditions.

(a) The population is increasing at a continuous rate of 6% per year.

$$\frac{dP}{dt} = 0.06P$$

(b) The population is decreasing at a continuous rate of 3% per year.

$$\frac{dP}{dt} = -0.03P$$

(c) The population is increasing at a rate of 20 people per year.

$$\frac{dP}{dt} = 20$$

(d) The population is decreasing at a rate of 15 people per year.

$$\frac{dP}{dt} = -15$$

(e) The population is growing logistically with an intrinsic growth rate of 3% per year and a carrying capacity of 800.

$$\frac{dP}{dt} = 0.03P\left(1 - \frac{P}{800}\right)$$

(f) The population is growing at a rate which is proportional to the square root of the population size with a constant of proportionality of 0.08.

$$\frac{dP}{dt} = 0.08\sqrt{P}$$

9. (5 points) Given that $\frac{dP}{dt} = 0.3P$ and $P(0) = 200$, find an explicit formula for P.

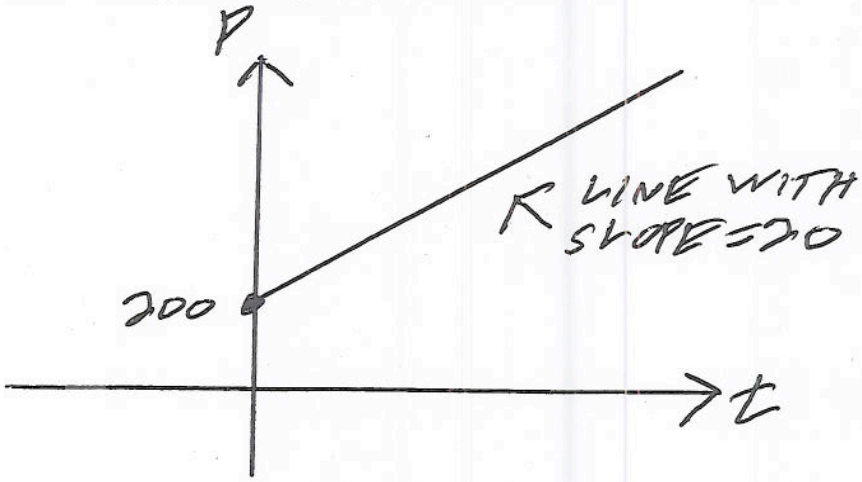
$P = 200e^{0.3t}$

10. (5 points) Given that $\frac{dP}{dt} = 6$ and $P(0) = 300$, find an explicit formula for P.

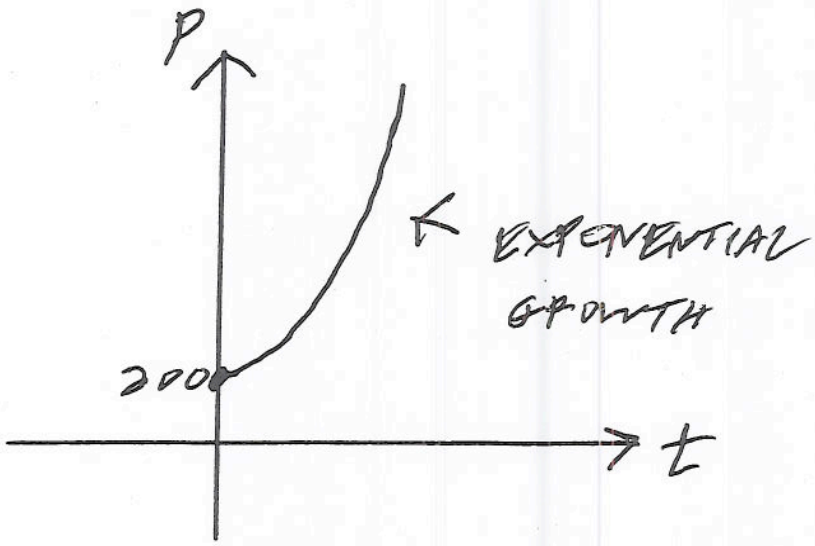
$P = 6t + 300$

11. (5 points each) Let P represent a town's population t years from now. Suppose that the current population is 200. Sketch a plausible graph for P if its growth is modeled by the given differential equation. Your graph should include all known coordinates for intercepts and inflection points, and should clearly show any long term behavior.

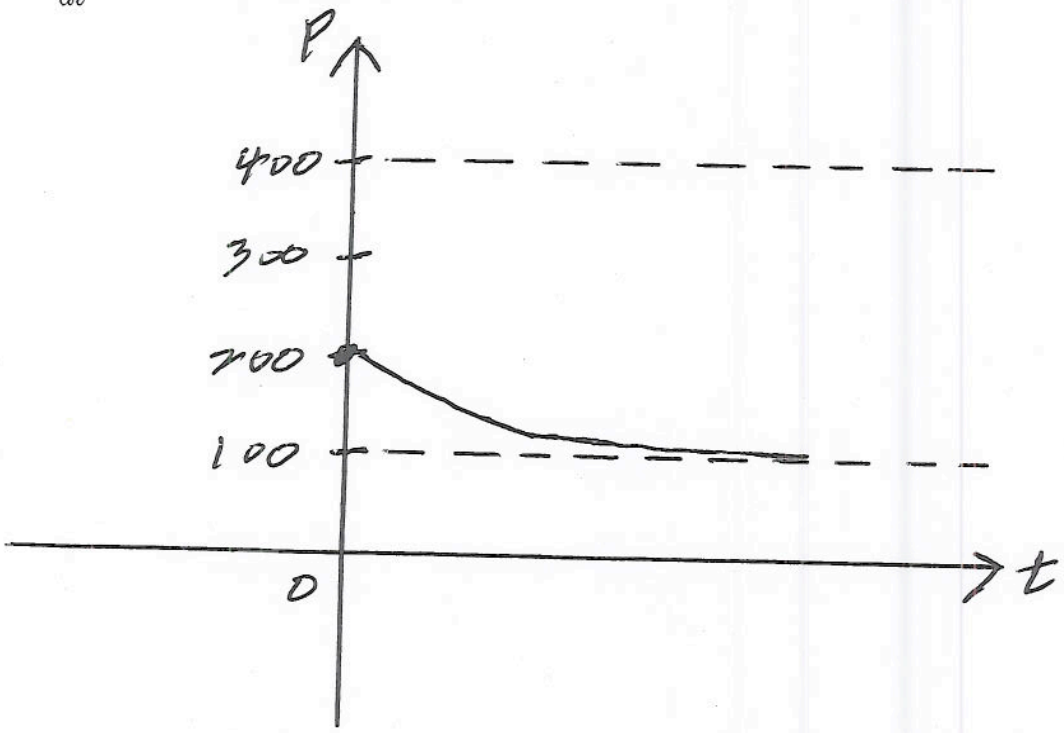
(a) $\frac{dP}{dt} = 20$



(b) $\frac{dP}{dt} = 0.03P$



(c) $\frac{dP}{dt} = 0.07(P - 100)(P - 400)$



(d) $\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{800}\right)$