

Name _____

Seat # _____

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your Student ID when you turn in your test.

Do not write below this line

#1 (5 points) _____

#8d (5 points) _____

#2 (5 points) _____

#8e (5 points) _____

#3 (5 points) _____

#8f (5 points) _____

#4 (5 points) _____

#9 (5 points) _____

#5 (5 points) _____

#10 (5 points) _____

#6 (5 points) _____

#11a (5 points) _____

#7 (10 points) _____

#11b (5 points) _____

#8a (5 points) _____

#11c (5 points) _____

#8b (5 points) _____

#11d (5 points) _____

#8c (5 points) _____

TOTAL (100 points) _____

1. (5 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 40$ and $\frac{dP}{dt} = 0.1P$?
- (a) P grows linearly by 10 people per year.
 - (b) P grows linearly by 40 people per year.
 - (c) P grows linearly by 60 people per year.
 - (d) P grows linearly by 90 people per year.
 - (e) P grows exponentially with a continuous growth rate of 10% per year.
 - (f) P grows exponentially with a continuous growth rate of 40% per year.
 - (g) P grows exponentially with a continuous growth rate of 60% per year.
 - (h) P grows exponentially with a continuous growth rate of 90% per year.
 - (i) P grows logistically with a carrying capacity of 10.
 - (j) P grows logistically with a carrying capacity of 40.
 - (k) P grows logistically with a carrying capacity of 60.
 - (l) P grows logistically with a carrying capacity of 90.
2. (5 points) Let P represent some population t years from now. Which one of the following statements is correct given that $P(0) = 40$ and $\frac{dP}{dt} = 10$?
- (a) P grows linearly by 10 people per year.
 - (b) P grows linearly by 40 people per year.
 - (c) P grows linearly by 60 people per year.
 - (d) P grows linearly by 90 people per year.
 - (e) P grows exponentially with a continuous growth rate of 10% per year.
 - (f) P grows exponentially with a continuous growth rate of 40% per year.
 - (g) P grows exponentially with a continuous growth rate of 70% per year.
 - (h) P grows exponentially with a continuous growth rate of 80% per year.
 - (i) P grows logistically with a carrying capacity of 10.
 - (j) P grows logistically with a carrying capacity of 40.
 - (k) P grows logistically with a carrying capacity of 70.
 - (l) P grows logistically with a carrying capacity of 80.

3. (5 points) It has been raining heavily since midnight. Let $f(t)$ represent the total number of inches of rain which have fallen in the t hours since midnight. Given that $f'(2) = 4$, which one of the following sentences must be true?
- (a) From midnight to 2:00AM it rained a total of 4 inches.
 - (b) From midnight to 4:00AM it rained a total of 2 inches.
 - (c) From midnight to 2:00AM it was raining at an average rate of 4 inches per hour.
 - (d) From midnight to 4:00AM it was raining at an average rate of 2 inches per hour.
 - (e) At 2:00AM it was raining at a rate of 4 inches per hour.
 - (f) At 4:00AM it was raining at a rate of 2 inches per hour.
4. (5 points) The maximum range of a projectile is proportional to the square of its velocity. A baseball pitcher can throw a ball at 60 miles per hour with a maximum range of 242 feet. What would his maximum range be if he could throw the ball at 65 miles per hour?
5. (5 points) A model for the population of a town predicts the population t years from now to be given by $P(t) = 400e^{-0.02t}$. How quickly in people per year is the population predicted to be changing 25 years from now?

6. (5 points) Find all equilibrium values for the following differential equation. There is no need to discuss whether or not these equilibrium values are stable.

$$\frac{dP}{dt} = 0.6(P^2 - 4)(P^2 + 9)(P - 15)$$

7. (10 points) Suppose P is a function of t whose growth is determined by the differential equation with initial condition shown. Use Euler's Method with $\Delta t = 3$ to approximate $P(9)$. Each step in your calculation should be correctly rounded off to three places after the decimal point.

$$\frac{dP}{dt} = \frac{8}{P^2} \quad \text{and} \quad P(0) = 5$$

8. (5 points each) Let P represent a town's population t years from now. Give a differential equation which models the population under the following conditions.
- (a) The population is increasing at a continuous rate of 6% per year.

 - (b) The population is decreasing at a continuous rate of 3% per year.

 - (c) The population is increasing at a rate of 20 people per year.

 - (d) The population is decreasing at a rate of 15 people per year.

 - (e) The population is growing logistically with an intrinsic growth rate of 3% per year and a carrying capacity of 800.

 - (f) The population is growing at a rate which is proportional to the square root of the population size with a constant of proportionality of 0.08.

9. (5 points) Given that $\frac{dP}{dt} = 0.3P$ and $P(0) = 200$, find an explicit formula for P .

10. (5 points) Given that $\frac{dP}{dt} = 6$ and $P(0) = 300$, find an explicit formula for P .

11. (5 points each) Let P represent a town's population t years from now. Suppose that the current population is 200. Sketch a plausible graph for P if its growth is modeled by the given differential equation. Your graph should include all known coordinates for intercepts and inflection points, and should clearly show any long term behavior.

(a) $\frac{dP}{dt} = 20$

(b) $\frac{dP}{dt} = 0.03P$

$$(c) \frac{dP}{dt} = 0.07(P - 100)(P - 400)$$

$$(d) \frac{dP}{dt} = 0.05P \left(1 - \frac{P}{800}\right)$$