Math 172 (Section 1) Test 3 Spring 2008

Name SOLUTIONS

Seat # ______

- Do not open this test booklet until told to do so.
- Turn off all cell phones.
- For multiple-choice questions, precisely one answer is correct. Circle this correct answer.
- For all other questions, you must show sufficient work to justify your answer.
- You are not allowed to borrow another student's calculator during the test.
- Show your Student ID when you turn in your test.

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**Do not write below this line**

| | | | |
|---|---|---|---|
| #1 (5 points) | ______ | #7 (15 points) | ______ |
| #2 (5 points) | ______ | #8 (15 points) | ______ |
| #3 (5 points) | ______ | #9 (15 points) | ______ |
| #4 (5 points) | ______ | #10 (5 points) | ______ |
| #5 (5 points) | ______ | #11 (10 points) | ______ |
| #6 (10 points) | ______ | #12 (5 points) | ______ |
| | | TOTAL (100 points) | ______ |

Test 1 ______ Test 2 ______ Test 3 ______ Total ______

If you skip the final exam, your course grade will be ______

1. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dq}{dv} = 3 \qquad \text{and} \qquad q(0) = 60$$

$q = 3v + C$

$60 = 3(0) + C \Rightarrow C = 60$

$q = 3v + 60$

2. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dp}{dt} = 10t \qquad \text{and} \qquad p(0) = 20$$

$p = 5t^2 + C$

$20 = 5(0)^2 + C \Rightarrow C = 20$

$p = 5t^2 + 20$

3. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dw}{dr} = 2w \qquad \text{and} \qquad w(0) = 30$$

$w = 30e^{2r}$

4. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dy}{dx} = -8e^{-2x} \qquad \text{and} \qquad y(0) = 25$$

$$y = 4e^{-2x} + c$$

$$25 = 4e^{-2(0)} + c$$

$$25 = 4 + c$$

$$c = 21$$

$$y = 4e^{-2x} + 21$$

5. (5 points) Find an explicit solution to the initial value problem.

$$\frac{dh}{dt} = \frac{e^t}{3h^2} \qquad \text{and} \qquad h(0) = 2$$

$$3h^2\, dh = e^t\, dt$$

$$\int 3h^2\, dh = \int e^t\, dt$$

$$h^3 = e^t + c$$

$$2^3 = e^0 + c$$

$$8 = 1 + c \Rightarrow c = 7$$

$$h^3 = e^t + 7$$

$$h = \sqrt[3]{e^t + 7}$$

6. (10 points) Suppose that a fish population grows logistically with an intrinsic growth rate of 25% and a carrying capacity of 800.

(a) Determine a discrete dynamical system to model this fish population.

$$P(t) = P(t-1) + 0.25P(t-1)\left(1 - \frac{P(t-1)}{800}\right)$$

OR

$$P(t) = P(t-1) + R \cdot P(t-1)$$

$$P(t) = P(t-1) + \left(\frac{-0.25}{800}P(t-1) + 0.25\right)P(t-1)$$

(b) Determine the maximum interval of stability for this fish population.

TRY VARIOUS INITIAL VALUES ON CALCULATOR OR SOLVE

$$0 = P + 0.25P\left(1 - \frac{P}{800}\right)$$

$$0 = P + 0.25P - \frac{0.25}{800}P^2$$

$$0 = P\left(1.25 - \frac{0.25}{800}P\right)$$

$$P = 0 \quad \text{OR} \quad \underbrace{1.25 - \frac{0.25}{800}P = 0}_{P = 4000}$$

MAX INTERVAL OF STABILITY IS $(0, 4000)$

7. (15 points) For John's metabolism, the dynamical system modeling the elimination of alcohol is given by

$$a(n) = a(n-1) - \frac{10a(n-1)}{4 + a(n-1)} + d$$

where $a(n)$ is the amount of alcohol (in grams) in his bloodstream after $n$ hours of drinking $d$ grams of alcohol per hour.

(a) If John were to drink 28 grams of alcohol per hour, how much alcohol would be in his bloodstream after drinking this amount for 4 hours? Begin with $a(0) = 0$.

| $n$ | $a(n)$ |
|---|---|
| 0 | 0 |
| 1 | 28 |
| 2 | 47.25 |
| 3 | 66.03 |
| 4 | 84.602 |

84.6 grams

(b) How many grams of alcohol per hour can John drink if at the end of a 4 hour party he is to have 40 grams of alcohol in his bloodstream? Begin with $a(0) = 0$ and give your answer correct to one place after the decimal.

TRYING VARIOUS VALUES FOR $d$ AND USING THE CALCULATOR'S TABLE OF VALUES, WE FIND $d$ IS BETWEEN 16.38 AND 16.39 grams.

I WILL ACCEPT EITHER 16.3 OR 16.4 grams

(c) Compute the equilibrium amount of alcohol in John's bloodstream if he drinks 9.5 grams of alcohol per hour.

$$a^* = a^* - \frac{10a^*}{4+a^*} + 9.5$$

$$\frac{10a^*}{4+a^*} = 9.5$$

$$10a^* = 9.5(4+a^*) = 38 + 9.5a^*$$

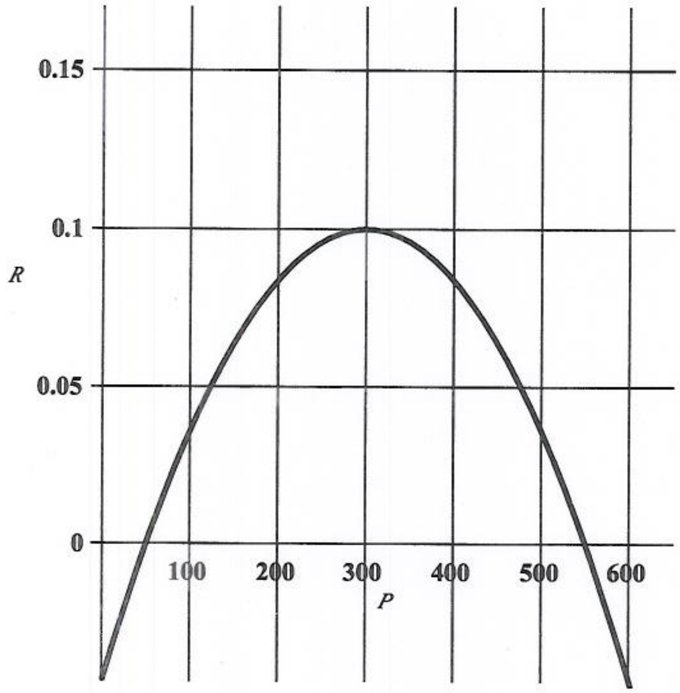
$$0.5a^* = 38$$

$$a^* = 76 \text{ grams}$$

8. (15 points) A population $P$ can be modeled by the discrete dynamical system

$$P(t) = P(t-1) + R \cdot P(t-1)$$

where the growth rate $R$ is a function of $P$ as shown in the graph below.




(a) Which one of the following is the intrinsic growth rate for this population?

$(A)$ 1% $(B)$ 3.33% $(C)$ 5% $(D)$ 10% $(E)$ 15% $(F)$ 30%

☆ FULL CREDIT FOR (A), (E) OR BOTH

(b) Which one of the following populations is a stable equilibrium value?

THE ANSWER I WAS EXPECTING

$(A)$ 0 $(B)$ 50 $(C)$ 100 $(D)$ 300 $(E)$ 550 $(F)$ 600

I MISTAKENLY INCLUDED THIS ANSWER, WHICH IS ALSO A CORRECT CHOICE

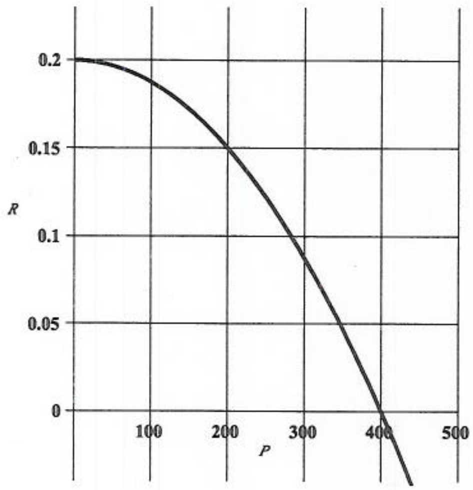
(c) Which one of the following is the minimum viable population?

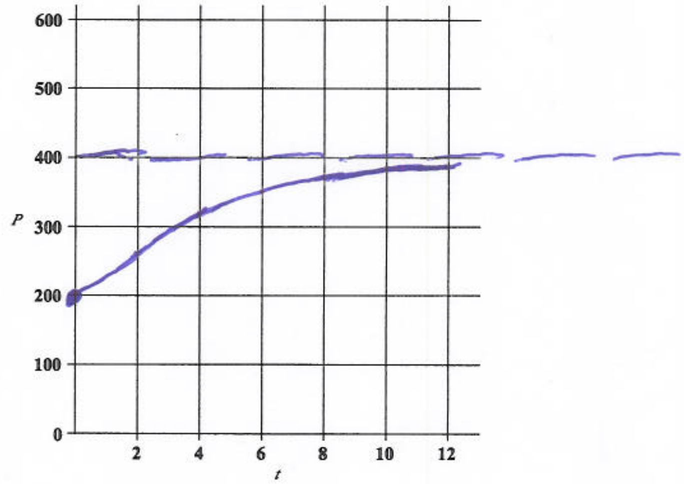
$(A)$ 0 $(B)$ 50 $(C)$ 100 $(D)$ 300 $(E)$ 550 $(F)$ 600

9. (15 points) A population $P$ can be modeled by the discrete dynamical system

$$P(t) = P(t-1) + R \cdot P(t-1)$$

where the growth rate $R$ is a function of $P$ as shown in the graph on the left.




(a) Given that $P(0) = 200$, use the grid on the right to sketch a rough graph of the population as a function of time. The time scale shown is sufficiently long that any long term behavior for the population should be apparent from your graph.

(b) Determine a formula for $R$ in terms of $P$ given that the graph of $R$ is a parabola with vertex $(0, 0.2)$.

$R = a(P-h)^2 + K$

$R = a(P-0)^2 + 0.2$

$R = aP^2 + 0.2$

$0 = a(400)^2 + 0.2$

$a = \dfrac{-0.2}{400^2}$

$R = \dfrac{-0.2}{400^2} P^2 + 0.2$

OR $R = 0.00000125 P^2 + 0.2$

(c) Compute the value of $P(1)$ given that $P(0) = 200$.

$P(1) = P(0) + R \cdot P(0)$

$P(1) = 200 + 0.15(200)$

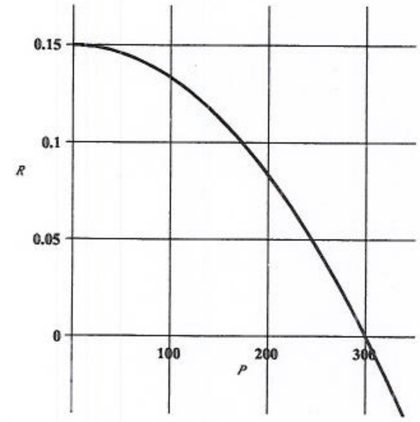
$P(1) = 230$

OR ENTER DYNAMICAL SYSTEM ON CALCULATOR TO GET $P(1) = 230$

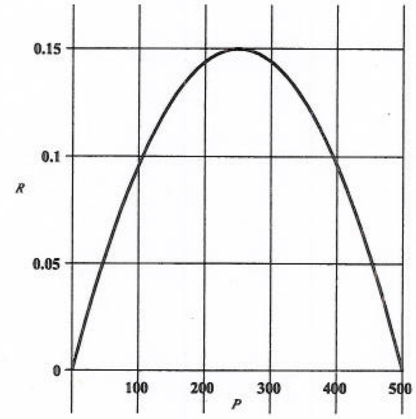
10. (5 points) A population grows logistically and is modeled by the following discrete dynamical system

$$P(t) = P(t-1) + R \cdot P(t-1)$$

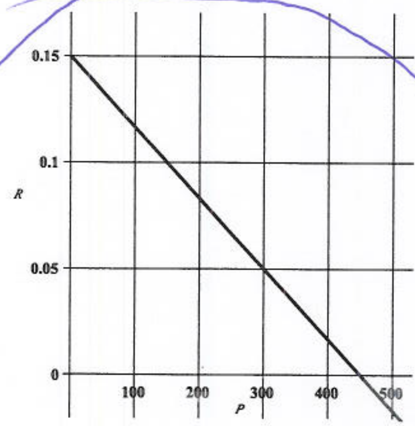
where $R$ represents the growth rate and is a function of the population $P$. Which one of the following graphs is the only one that could possibly be the graph of $R$ as a function of $P$ ?



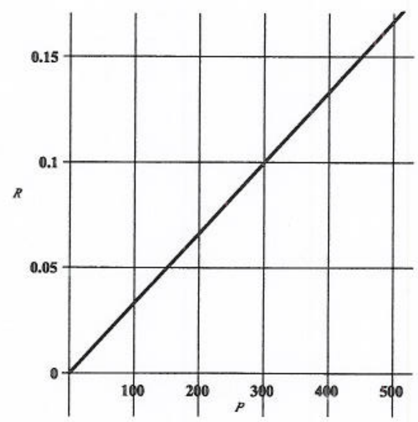

$(a)$



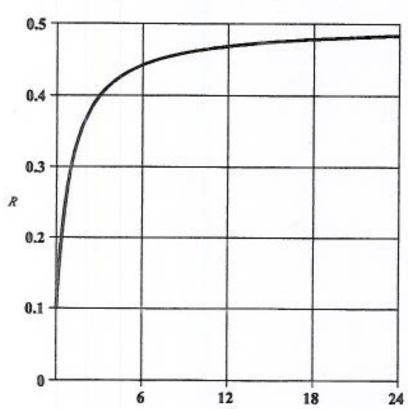

$(b)$



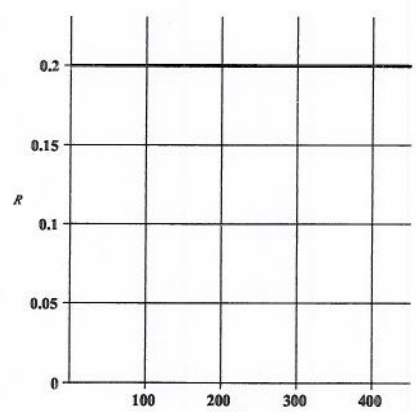

$(c)$




$(d)$

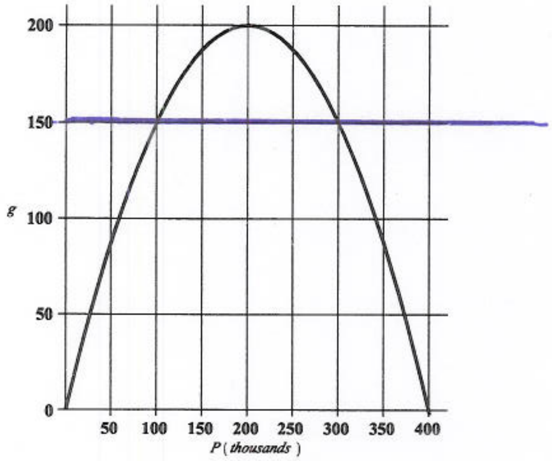



$(e)$




$(f)$

11. (10 points) The natural yearly growth $g$ in a fish population is a function of the population size $P$ (in thousands) and is shown in the following graph. Suppose that there is a constant yearly harvest of 150 fish.




$h = 150$

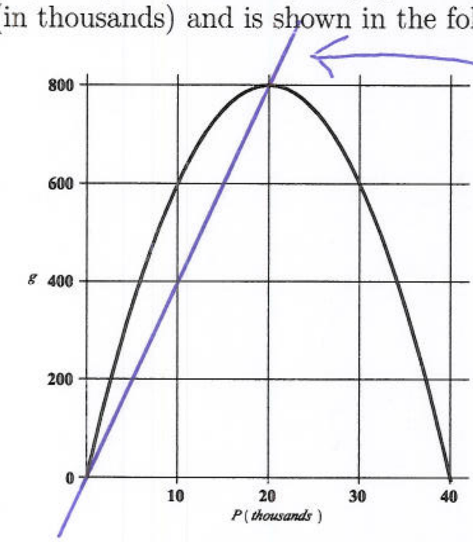
(a) Estimate the stable equilibrium population.

300,000 fish

(b) Estimate the minimum viable population.

100,000 fish

12. (5 points) The natural yearly growth $g$ in a population is a function of the population size $P$ (in thousands) and is shown in the following graph.




$h = rP$

$800 = r(20000)$

$r = \frac{800}{20000} = 0.04$

What percent of the population should be harvested each year if you wish to maximize the eventual yearly harvest?

$h = 0.04P$ 4%