Math 172 Quiz 1 January 24, 2008

Name SOLUTIONS

1. Suppose that W is a function of t and takes on the values shown in the following table. Determine a discrete dynamical system along with an intial value for this function. Determine an explicit formula for this function.

2 points

t	W
0	4
1	9
2	14
3	19
4	24

discrete dynamical system

$w(t+1) = w(t) + 5$
$w(0) = 4$

or

$w(t) = w(t-1) + 5$
$w(0) = 4$

explicit formula

$w(t) = 5t + 4$

2. Given that $P(0) = 30$ and $P(n+1) = 0.9P(n) + 45$, make a table of values for P with $n = 0, 4, 8, 12, 16,$ and 20. If you observe the behavior of this function for larger values of n, what long term behavior do you observe?

2 points

n	$P(n)$
0	30
4	174.44
8	269.2
12	331.38
16	372.17
20	398.94
⋮	⋮
92	449.97
96	449.98
100	449.99
104	449.99

P increases and levels off at 450

3 points

3. Suppose that a patient takes a daily dose of 2.5 milligrams of some drug, and that each day the kidneys filter out 8% of this drug from the patient's bloodstream.

(a) Determine a discrete dynamical system along with an initial value for the number of milligrams of this drug in this patient's bloodstream each day. State precisely what each variable represents in this system.

Let $D(t)$ be the number of milligrams of this drug in the bloodstream t days after the initial dose.

$$D(t) = D(t-1) - 0.08D(t-1) + 2.5$$

$$D(t) = 0.92D(t-1) + 2.5$$
$$D(0) = 2.5$$

(b) How many milligrams of this drug are in the bloodstream 5 days after the initial dose?

t	$D(t)$
0	2.5
1	4.8
2	6.916
3	8.8627
4	10.654
5	12.301

$D(5) \approx 12.3$ mg

(c) Assuming that the patient is to continue taking this drug for a long time, what was the doctor's **target goal** for the desired amount of this drug in the bloodstream?

t	$D(t)$
0	2.5
20	25.825
40	30.226
60	31.057
80	31.214
100	31.243
120	31.249
140	31.250
160	31.250

31.25 mg

3 points

4. In 1960 there were 500 people living in a small town. Suppose that the population increased by 12% each year since then. Let $P(t)$ represent the population of the town t years after 1960.

   (a) Find a discrete dynamical system along with an initial value for $P(t)$.

   $P(t) = P(t-1) + 0.12P(t-1)$

   $P(t) = 1.12P(t-1)$

   $P(0) = 500$

   (b) Find an explicit formula for $P(t)$.

   $P(t) = 500(1.12)^t$

   (c) In what year will the population reach 1500 people?

   $1500 = 500(1.12)^t$

   $3 = 1.12^t$

   $\ln 3 = \ln(1.12^t)$

   $\ln 3 = t \cdot \ln(1.12)$

   $t = \ln 3 / \ln(1.12)$

   $t \approx$ 9.694 years after the 1960 census

   or use a table to approximate.